

**LESSON**  
**3·1**
**General Patterns and Special Cases**


1. You are describing a general number pattern for a special case when you write a rule for a “What’s My Rule?” table.

Write a rule for each table shown below.

in	out
8	13
11	16
20	25
105	110

in	out
12	4
21	7
60	20
300	100

Rule: \_\_\_\_\_

Rule: \_\_\_\_\_

2. You are writing special cases for a general number pattern when you complete a “What’s My Rule?” table.

Complete.

Rule: Add the opposite of the number.

$$(x + -x = 0)$$

in	out
3	0
25	
-7	
-53	

Rule: Divide by the number.

$$(y \div y = 1)$$

in	out
8	1
9	
$\frac{1}{4}$	
100	

Use the values from the table above to write special cases for the following general number patterns:

$$x + -x = 0.$$

Special cases

**Example:**  $3 + -3 = 0$

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

$$y \div y = 1.$$

Special cases

**Example:**  $8 \div 8 = 1$

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

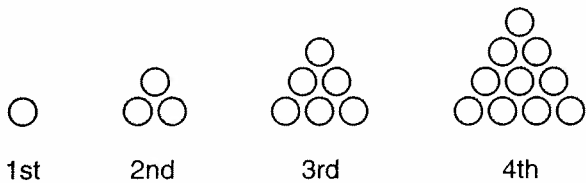
**LESSON**  
**3·1**

# Number Patterns

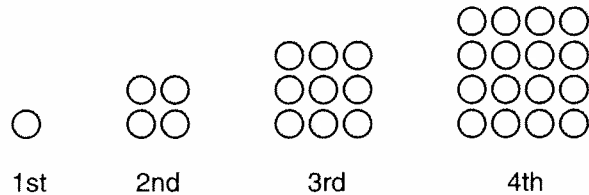


Triangular, square, and rectangular numbers are examples of number patterns that can be shown by geometric arrangements of dots. Study the number patterns shown below.

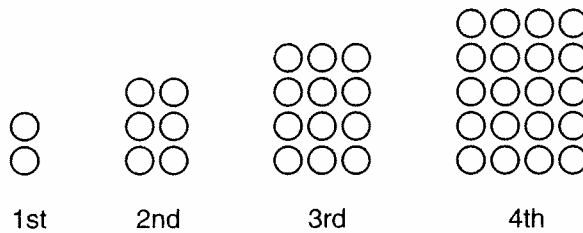
### Triangular Numbers



### Square Numbers



### Rectangular Numbers



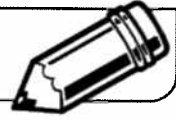
1. Use the number patterns to complete the table.

Number of Dots in Arrangement										
	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
<b>Triangular Number</b>	1	3	6	10						
<b>Square Number</b>	1	4	9	16						
<b>Rectangular Number</b>	2	6	12	20						

2. What is the 11th triangular number? \_\_\_\_\_

How does the 11th triangular number compare to the 10th triangular number?

\_\_\_\_\_

**LESSON**  
**3•1**
**Number Patterns** *continued*


3. Describe what you notice about the sum of 2 triangular numbers that are next to each other in the table.

---



---

4. Add the second square number and the second rectangular number; the third square number and the third rectangular number. What do you notice about the sum of a square number and its corresponding rectangular number?

---



---

5. Describe any other patterns you notice.

---



---

6. You can write triangular numbers as the sum of 4 triangular numbers when repetitions are allowed. For example:  $6 = 1 + 1 + 1 + 3$

Find 3 other triangular numbers that can be written as sums of *exactly* 4 triangular numbers.

$$\underline{\quad} = \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad}$$

$$\underline{\quad} = \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad}$$

$$\underline{\quad} = \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad}$$

**STUDY LINK**  
**3•1**

# Variables in Number Patterns



1. Following are 3 special cases representing a general pattern.

$17 + 0 = 17$

$-43 + 0 = -43$

$\frac{7}{8} + 0 = \frac{7}{8}$

a. Describe the general pattern in words.

---



---

b. Give 2 other special cases for the pattern.

---

For each general pattern, give 2 special cases.

2.  $(2 * m) + m = 3 * m$

---

3.  $s + 0.25 = 0.25 + s$

---

For each set of special cases, write a general pattern.

4.  $3^2 * 3^3 = 3^5$

5.  $7 * 0.1 = \frac{7}{10}$

6.  $2^0 = 1$

$5^2 * 5^3 = 5^5$

$3 * 0.1 = \frac{3}{10}$

$146^0 = 1$

$13^2 * 13^3 = 13^5$

$4 * 0.1 = \frac{4}{10}$

$\left(\frac{1}{2}\right)^0 = 1$

---

## Practice

Complete.

7.  $\frac{1}{10} = \frac{\boxed{\phantom{00}}}{100} = 0.10$

8.  $\frac{1}{4} = \frac{25}{\boxed{\phantom{00}}} = 0.\boxed{\phantom{00}}$

9.  $\frac{1}{5} = \frac{\boxed{\phantom{00}}}{100} = 0.20$

10.  $\frac{3}{4} = \frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}} = 0.75$

11.  $\frac{4}{5} = \frac{\boxed{\phantom{00}}}{100} = 0.\boxed{\phantom{00}}$

12.  $\frac{7}{10} = \frac{\boxed{\phantom{00}}}{100} = 0.\boxed{\phantom{00}}$

**STUDY LINK**  
**3•2**

# General Patterns with Two Variables



For each general pattern, write 2 special cases.

1.  $(6 * b) * c = 6 * (b * c)$

\_\_\_\_\_

\_\_\_\_\_

2.  $a \div \frac{b}{2} = (2 * a) \div b$

\_\_\_\_\_

\_\_\_\_\_

3.  $\frac{x}{y} = x * \frac{1}{y}$

(y is not 0)

\_\_\_\_\_

\_\_\_\_\_

For each set of special cases, write a number sentence with 2 variables to describe the general pattern.

4.  $7 - 5 = 7 + (-5)$

$12 - 8 = 12 + (-8)$

$9 - 1 = 9 + (-1)$

General pattern:

\_\_\_\_\_

5.  $\frac{4}{6} = \frac{4 * 3}{6 * 3}$

$\frac{1}{2} = \frac{1 * 3}{2 * 3}$

$\frac{2}{5} = \frac{2 * 3}{5 * 3}$

General pattern:

\_\_\_\_\_

6.  $\frac{6}{10} = \frac{6 \div 2}{10 \div 2}$

$\frac{4}{12} = \frac{4 \div 2}{12 \div 2}$

$\frac{2}{4} = \frac{2 \div 2}{4 \div 2}$

General pattern:

\_\_\_\_\_

7.  $\frac{1}{5} * \frac{1}{2} = \frac{1 * 1}{5 * 2}$

$\frac{2}{3} * \frac{1}{2} = \frac{2 * 1}{3 * 2}$

$\frac{3}{4} * \frac{1}{2} = \frac{3 * 1}{4 * 2}$

General pattern:

\_\_\_\_\_

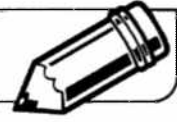
## Practice

Write each fraction as a decimal.

8.  $\frac{250}{100} =$  \_\_\_\_\_

9.  $\frac{106}{100} =$  \_\_\_\_\_

10.  $\frac{100}{100} =$  \_\_\_\_\_

**LESSON**  
**3•2**
**True and Not True Special Cases**


For each of the following, write one special case for which the sentence is true. Then write one special case for which the sentence is not true.

1.  $m * n = m + n$

True \_\_\_\_\_

Not true \_\_\_\_\_

2.  $\frac{a}{2} + b = a + b$

True \_\_\_\_\_

Not true \_\_\_\_\_

For each of the following, write at least 2 special cases for which the sentence is true. Circle each sentence that you think expresses a general pattern that is always true.

3.  $a^2 = 2 * a$

\_\_\_\_\_  
 \_\_\_\_\_

4. If  $a$  is not 0, then  $\frac{a^m}{a^n} = a^{m-n}$

\_\_\_\_\_  
 \_\_\_\_\_

5.  $(a + b) * (a - b) = a^2 - b^2$

\_\_\_\_\_  
 \_\_\_\_\_

**STUDY LINK**  
**3•3**
**General Patterns with Two Variables**


Write an algebraic expression for each situation. Use the suggested variable.

- Kayla has  $x$  CDs in her music collection. If Miriam has 7 fewer CDs than Kayla, how many CDs does Miriam have? \_\_\_\_\_ CDs
- Chaz ran 2.5 miles more than Nigel. If Nigel ran  $d$  miles, how far did Chaz run? \_\_\_\_\_ miles
- If a car dealer sells  $c$  automobiles each year, what is the average number of automobiles sold each month?  
\_\_\_\_\_ automobiles

First translate each situation from words into an algebraic expression. Then solve the problem that follows.

- The base of a rectangle is twice the length of the height. If the height of the rectangle is  $h$  inches, what is the length of the base?  
\_\_\_\_\_ inches

If the height of the rectangle is 4 inches, what is the length of the base?

\_\_\_\_\_ inches

**Try This**

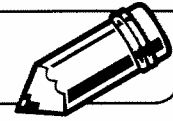
- Monica has 8 more than 3 times the number of marbles Regina has. If Regina has  $r$  marbles, how many marbles does Monica have?

\_\_\_\_\_ marbles

If Regina has 12 marbles, how many does Monica have? \_\_\_\_\_ marbles

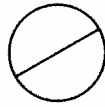
**Practice**

- $2.75 \text{ m} =$  \_\_\_\_\_  $\text{cm}$
- $3.5 \text{ cm} =$  \_\_\_\_\_  $\text{mm}$
- $500 \text{ m} =$  \_\_\_\_\_  $\text{km}$

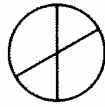
**LESSON**  
**3•3**
**“What’s My Rule?” for Geometric Patterns**


1. When you cut a circular pizza, each cut goes through the center.

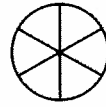
Cuts	Pieces
1	2
2	4
3	
4	
	12
	16



1 cut



2 cuts



3 cuts

Fill in the missing numbers in the table. Then write an algebraic expression that describes how many pieces you have when you make  $c$  cuts.

\_\_\_\_\_ pieces

2. Fold a sheet of paper in half. Now fold it in half again. And again. And again, until you can't make another fold.

After each fold, count the number of rectangles into which the paper has been divided. Fill in the missing numbers in the table. Write an algebraic expression to name the number of rectangles you have after you have folded the paper  $k$  times.

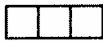
\_\_\_\_\_ rectangles

Folds	Rectangles
0	1
1	2
2	
3	
4	
5	
6	

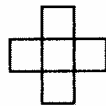
3. Below are the first 4 designs in a pattern made with square blocks. Draw Design 5 in this pattern.



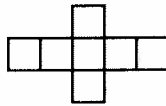
Design 1



Design 2



Design 3



Design 4

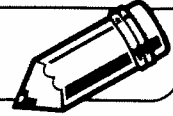
Design 5

4. How many square blocks will there be in

a. Design 10? \_\_\_\_\_

b. Design  $n$ ? \_\_\_\_\_



**LESSON**  
**3•3****More Algebraic Expressions**

Write each word phrase as an algebraic expression.

1.  $t$  increased by 5 \_\_\_\_\_
2. the product of  $w$  and 3 \_\_\_\_\_
3. 7 less than  $g$  \_\_\_\_\_
4.  $m$  halved \_\_\_\_\_
5.  $k$  shared equally by 8 people \_\_\_\_\_
6. 24 less than  $x$  tripled \_\_\_\_\_
7.  $b$  decreased by 12 \_\_\_\_\_

Evaluate each expression when  $y = 9.05$ .

8.  $y + 4.98$  \_\_\_\_\_      9.  $y - 8.9$  \_\_\_\_\_      10.  $y * 10^2$  \_\_\_\_\_

Write an algebraic expression for each situation. Then solve the problem that follows.

11. Talia earns  $d$  dollars per week.

How much does Talia earn in 10 weeks? \_\_\_\_\_ dollars

If Talia earns \$625.75 per week,

how much does she earn in 10 weeks? \_\_\_\_\_ dollars

12. Michelle is 5 years younger than Ruby, who is  $r$  years old. Kyle is twice as old as Michelle.

- a. Using Ruby's age,  $r$ , write an expression for:

Michelle's age \_\_\_\_\_ years old

Kyle's age \_\_\_\_\_ years old

- b. Suppose Ruby is 12 years old. Find:

Michelle's age \_\_\_\_\_ years old

Kyle's age \_\_\_\_\_ years old

**STUDY LINK**  
**3•4**

# “What’s My Rule?” Part 1



1. a. State in words the rule for the “What’s My Rule?” table at the right.

$m$	$n$
4.56	4.34
10	9.78
0.01	-0.21
$\frac{24}{100}$	0.02
7.80	7.58

- b. Which formula describes the rule? Fill in the circle next to the best answer.

(A)  $n = m - 0.22$      
  (B)  $m + n = 0.22$      
  (C)  $m = n - 0.22$

2. a. State in words the rule for the “What’s My Rule?” table at the right.

$r$	$t$
20	10
15	7.5
1	0.5
1.5	0.75
3.4	1.7

- b. Which formula describes the rule? Fill in the circle next to the best answer.

(A)  $r - 0.25 = t$      
  (B)  $t + 0.12 = r$      
  (C)  $r * 0.5 = t$

3. Which formula describes the rule for the “What’s My Rule?” table at the right? Fill in the circle next to the best answer.

$p$	$q$
7	12
10	18
1	0
15	28
30	58

(A)  $q - 13 = p$      
  (B)  $q = (2 * p) - 2$      
  (C)  $q = 2 * (p - 2)$

**Practice**

4. 180 in. = \_\_\_\_\_ feet     
 5.  $3\frac{1}{2}$  minutes = \_\_\_\_\_ seconds  
 6. 5,280 ft = \_\_\_\_\_ yards     
 7.  $5\frac{1}{2}$  miles = \_\_\_\_\_ feet

**LESSON**  
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# Special Cases for Formulas

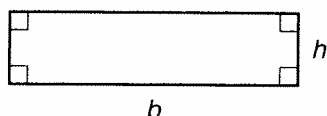


A formula is an example of a general pattern. When you substitute values for the variables in a formula, you are writing a special case for the formula.

## Area Formulas

### Example:

To find the area of a rectangle, use the formula  $A = b * h$ .



Write a special case for the formula using  $b = 12$  cm and  $h = 3$  cm.

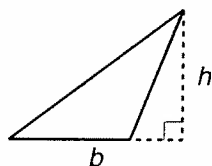
First substitute only the value of  $b$ .  $A = 12$  cm \*  $h$

Next substitute the value of  $h$ .  $A = 12$  cm \* **3 cm**

Now find the value of  $A$  and write the special case.

$$36 \text{ cm}^2 = 12 \text{ cm} * 3 \text{ cm}$$

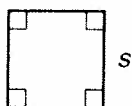
1. To find the area of a triangle, use the formula  $A = \frac{1}{2} * (b * h)$ .



Find the value of  $A$  and write a special case for the formula using  $b = 4.5$  cm and  $h = 4.8$  cm.

$$\underline{\hspace{2cm}} \text{ cm}^2 = \frac{1}{2} * (\underline{\hspace{2cm}} \text{ cm} * \underline{\hspace{2cm}} \text{ cm})$$

2. To find the area of a square, use the formula  $A = s^2$ .

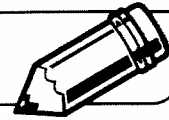


Find the value of  $A$  and write a special case for the formula using  $b = 2.5$  ft and  $h = 2.5$  ft.

$$\underline{\hspace{2cm}} \text{ ft}^2 = \underline{\hspace{2cm}} \text{ ft}^2$$

**LESSON**  
**3•4**

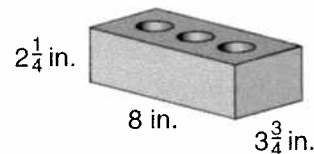
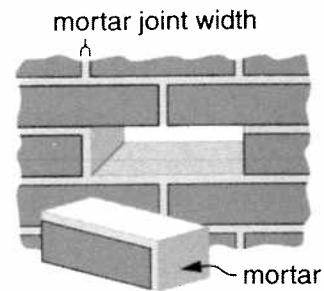
# Formula for a Brick Wall



Suppose you were going to build a brick wall. It would be useful to estimate the number of bricks you would need. You could do this if you had a formula for estimating the number of bricks for any size wall.

Study the following information. Then follow the instructions for measuring an actual brick wall. Try to devise a formula for estimating the number of bricks needed to build any size wall.

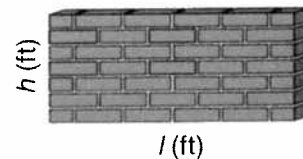
- ◆ A brick wall is built by putting layers of bricks on top of one another. The space between the bricks is filled with a material called *mortar*, which hardens and holds the bricks in place. The mortar between the bricks forms the *mortar joint*.
- ◆ A standard building brick is  $2\frac{1}{4}$  inches by 8 inches by  $3\frac{3}{4}$  inches. The face that is  $2\frac{1}{4}$  inches by 8 inches is the part of the brick that is visible in a wall.



Equivalents:  $1 \text{ ft} = 12 \text{ in.}$        $1 \text{ ft}^2 = 12 \text{ in.} * 12 \text{ in.} = 144 \text{ in.}^2$

1. Find a brick wall in your school, home, or neighborhood. Using a ruler or tape measure, measure the length and height of the wall or part of the wall. Count the bricks in the area you measured.
  - a. Length \_\_\_\_\_ (unit)
  - b. Height \_\_\_\_\_ (unit)
  - c. Number of bricks \_\_\_\_\_
  - d. Measure the width of the mortar joint in several places. Decide on a typical value for this measurement.

The mortar joints are each about \_\_\_\_\_ inch(es) wide.



2. Devise a formula for calculating the number of bricks needed to build a wall. Let  $l$  stand for the length of the wall in feet. Let  $h$  stand for the height of the wall in feet. Let  $N$  stand for the estimated number of bricks needed to build the wall.
  - a. The area of this wall (the side you see) is \_\_\_\_\_ square feet.
  - b. My formula for the estimated number of bricks:  $N =$  \_\_\_\_\_
3. Test your formula. Use the length and width you measured in Problem 1. Does the formula predict the number of bricks you counted?

**STUDY LINK**  
**3•5**

# “What’s My Rule?” Part 2



1. *Rule:* Subtract the *in* number from  $11\frac{1}{2}$ .

in	out
$n$	$11\frac{1}{2} - n$
1	
2	
$8\frac{1}{2}$	
	5
12	

2. *Formula:*  $r = 4 * s$

in	out
$s$	$r$
12	
	24
0.3	
	1
$\frac{1}{2}$	

3. *Rule:* Triple the *in* number and add  $-6$ .

in	out
$x$	$(3x) + (-6)$
1	$-3$
2	
	15
8	
	$-6$

4. For the table below, write the rule in words and as a formula.

*Rule:* \_\_\_\_\_  
 \_\_\_\_\_

*Formula:* \_\_\_\_\_

in	out
$b$	$d$
1.5	0.5
$6\frac{3}{4}$	$2\frac{1}{4}$
24	8
81	27
9.75	3.25

5. Make up your own.

*Rule:* \_\_\_\_\_  
 \_\_\_\_\_

*Formula:* \_\_\_\_\_

in	out
$x$	$y$

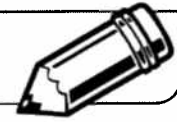
**Practice**

6.  $3 + -6 =$  \_\_\_\_\_

7.  $-17 + 5 =$  \_\_\_\_\_

8.  $8 + (-2) + (-9) =$  \_\_\_\_\_

9.  $5 + 3 + (-5) + 7 =$  \_\_\_\_\_

**LESSON**  
**3•5****Rates**

Solve the rate problems. You can use tables similar to “What’s My Rule?” tables to help you find the answers, if needed.

1. Renee reads 30 pages per hour.

Hours	Pages
1	30
2	
3	

- a. At this rate, how many pages can she read in 3 hours?

\_\_\_\_\_ (unit)

- b. Would she be able to read a 220-page book in 7 hours?

\_\_\_\_\_

2. Wilson was paid \$105 to cut 7 lawns. At this rate, how much was he paid per lawn?

Lawns	Dollars
7	105
6	
5	

\_\_\_\_\_

3. Gabriel blinks 80 times in 5 minutes.

Minutes	Blinks
5	80
4	
3	

- a. At this rate, how many times does he blink in 2 minutes?

\_\_\_\_\_ (unit)

- b. In 4 minutes?

\_\_\_\_\_ (unit)

4. Michael can bake 9 batches of cookies in 3 hours.

Hours	Batches
3	9
2	
1	

- At this rate, how many batches can he bake in 2 hours?

\_\_\_\_\_ (unit)

5. Elizabeth can run 5 miles in  $\frac{2}{3}$  of an hour.

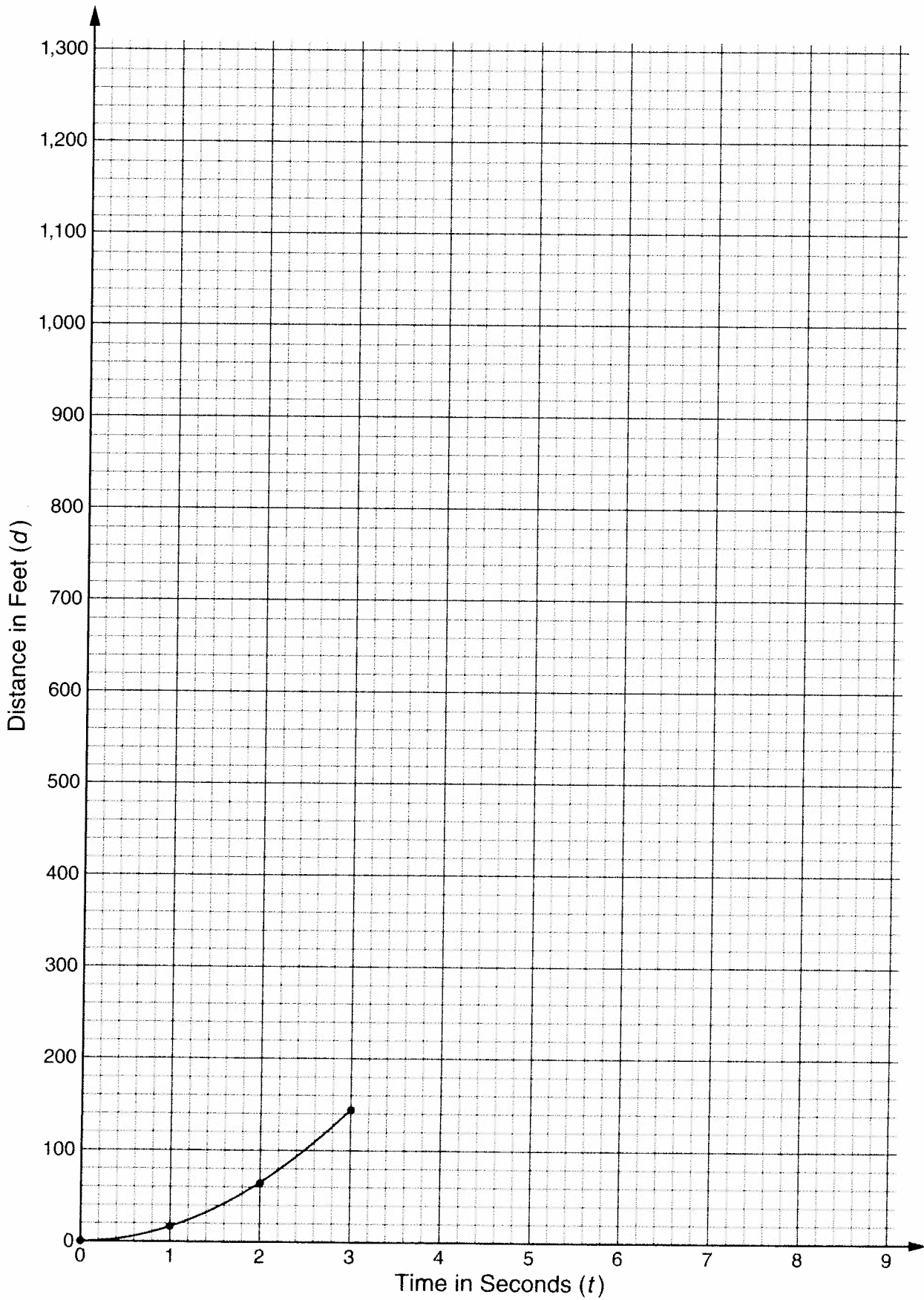
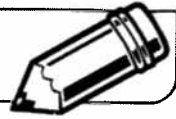
Hours	Miles
$\frac{2}{3}$	5
$\frac{1}{3}$	
1	

- At this rate, how long does it take her to run 1 mile?

\_\_\_\_\_ (unit)

**LESSON**  
**3•6**

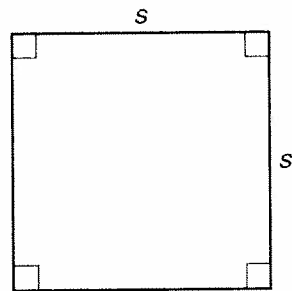
# When an Object Is Dropped



# Area and Perimeter

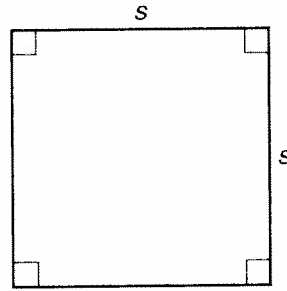


Perimeter



$$P = 4 * s$$

Area



$$A = s^2$$

1. Use the perimeter and area formulas for squares to complete the table.

Length of side (in.)	Perimeter (in.)	Area (in. <sup>2</sup> )
1		
2		
3		
4		
5		

Use the table above to complete the graphs on *Math Masters*, page 87.