

STUDY LINK
4·1

Equivalent Fractions



Find an equivalent fraction by multiplying.

1. $\frac{4}{5}$ _____

2. $\frac{7}{10}$ _____

3. $\frac{1}{4}$ _____

4. $\frac{2}{3}$ _____

5. $\frac{5}{4}$ _____

6. $\frac{2}{2}$ _____

Find an equivalent fraction by dividing.

7. $\frac{9}{12}$ _____

8. $\frac{20}{100}$ _____

9. $\frac{4}{16}$ _____

10. $\frac{30}{12}$ _____

11. $\frac{10}{50}$ _____

12. $\frac{16}{24}$ _____

Write 3 equivalent fractions for each number.

13. $\frac{1}{3}$ _____

14. $\frac{75}{100}$ _____

15. 6 _____

16. $\frac{12}{5}$ _____

Write each fraction in simplest form.

17. $\frac{8}{16}$ _____

18. $\frac{6}{9}$ _____

19. $\frac{3}{15}$ _____

20. $\frac{10}{25}$ _____

21. $\frac{6}{16}$ _____

22. $\frac{14}{49}$ _____

Find the missing numbers.

23. $\frac{1}{5} = \frac{x}{15}$

24. $\frac{2}{3} = \frac{y}{18}$

25. $\frac{15}{25} = \frac{m}{50}$

$x =$ _____

$y =$ _____

$m =$ _____

Practice

Divide. Express the remainder as a fraction in simplest form.

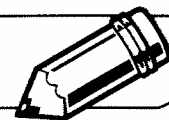
26. $24 \overline{)654}$

27. $25 \overline{)730}$

28. $14 \overline{)410}$

LESSON
4•1

Factor Rainbows



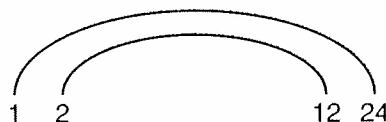
When listing the factors of a number, you need to be certain that you have included all the factors in your list. Creating a **factor rainbow** is one way to do this. A factor rainbow is an organized list of factor pairs.

Example: factor rainbow for 24

Every number is divisible by 1. Because $1 * 24 = 24$, use an arc to show that 1 and 24 are paired.



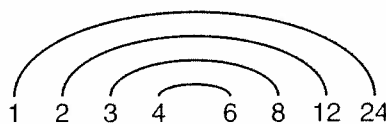
Now try dividing by 2. Because $2 * 12 = 24$, use an arc to pair 2 and 12.



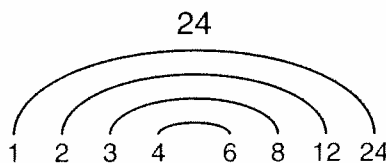
Continue your divisibility tests by moving to 3, which is the next factor greater than 2. 24 is divisible by 3, so $3 * 8 = 24$. Pair 3 and 8.



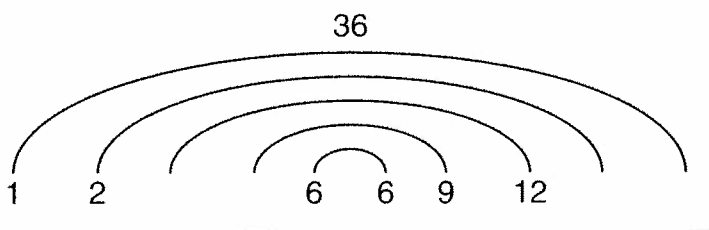
From the arcs you have drawn, you can see that all remaining factors must be between 3 and 8. Try dividing 24 by 4. Because $4 * 6 = 24$, use an arc to pair 4 and 6.

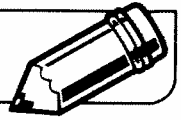


Any remaining factors must be between 4 and 6. The only whole number between 4 and 6 is 5. Notice that 5 does not divide into 24 evenly, so your rainbow is complete.



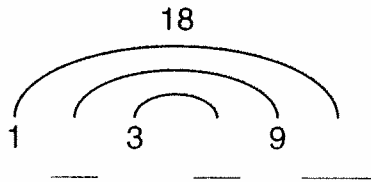
Use the example above when completing the factor rainbow for 36. Because 36 is a square number, one of the factors (6) is paired with itself.



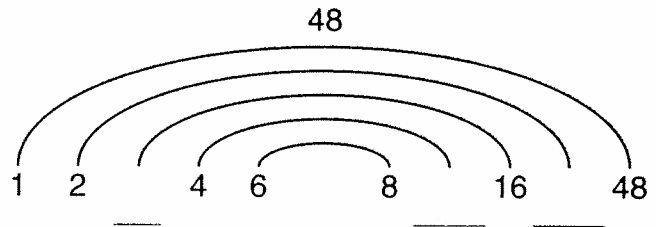
LESSON
4•1**Factor Rainbows** *continued*

Use the examples on page 106 to help you complete the factor rainbow for each number.

1. factor rainbow for 18



2. factor rainbow for 48

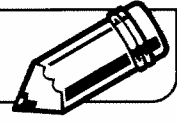


3. factor rainbow for 12

4. factor rainbow for 32

5. factor rainbow for 40

6. factor rainbow for 64

LESSON
4•1**Applications of the GCF**

Some real-world problems involve finding the greatest common factor (GCF) of a set of numbers.

Solve.

1. Tyrone is preparing snack packs for the class field trip. He has 60 bags of chips and 90 bottles of fruit juice. Each pack should have the same number of bags of chips and the same number of bottles of fruit juice. What is the greatest number of snack packs that Tyrone can make with no bags or bottles left over?

The greatest number of snack packs that he can make is _____.

Each snack pack will have _____ bags of chips and _____ bottles of fruit juice.

2. Carla has 30 blue beads, 60 red beads, and 72 white beads. What is the maximum number of friends to whom Carla can give the same number of beads and have no beads left over?

The maximum number of friends to get beads is _____.

Each friend will get _____ blue beads, _____ red beads, and _____ white beads.

3. Ms. Mendis wants to split her class into groups for a bridge-building contest. There are 32 students in her class. She has 16 bottles of wood glue, 1,200 craft sticks, and 24 jars of paint. What is the greatest number of groups that Ms. Mendis can make so each group gets the same number of supplies and no supplies are left over?

The greatest number of groups that she can make is _____.

Each group will get _____ bottles of wood glue, _____ craft sticks, and _____ jars of paint.

STUDY LINK
4·2

Comparing and Ordering Fractions



Write $<$, $>$, or $=$ to make a true number sentence. For each problem that you did not solve mentally, show how you got the answer.

1. $\frac{4}{5}$ _____ $\frac{2}{5}$

2. $\frac{3}{8}$ _____ $\frac{1}{3}$

3. $\frac{3}{4}$ _____ $\frac{17}{20}$

4. $\frac{19}{20}$ _____ $\frac{99}{100}$

5. $\frac{4}{7}$ _____ $\frac{4}{10}$

6. $\frac{2}{3}$ _____ $\frac{7}{9}$

7. Circle each fraction that is less than $\frac{1}{2}$. $\frac{3}{6}$ $\frac{6}{10}$ $\frac{1}{3}$ $\frac{2}{5}$ $\frac{6}{11}$ $\frac{12}{25}$

8. Write the fractions in order from smallest to largest.

$\frac{1}{3}$ $\frac{15}{16}$ $\frac{7}{14}$ $\frac{1}{5}$ $\frac{2}{5}$ $\frac{49}{50}$ $\frac{6}{10}$ $\frac{1}{12}$

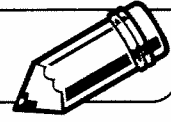
Practice

9. $13.987 - 4.09 =$ _____

10. $5.9 - 2.068 =$ _____

11. $0.9 - 0.077 =$ _____

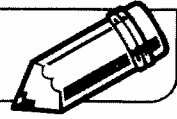
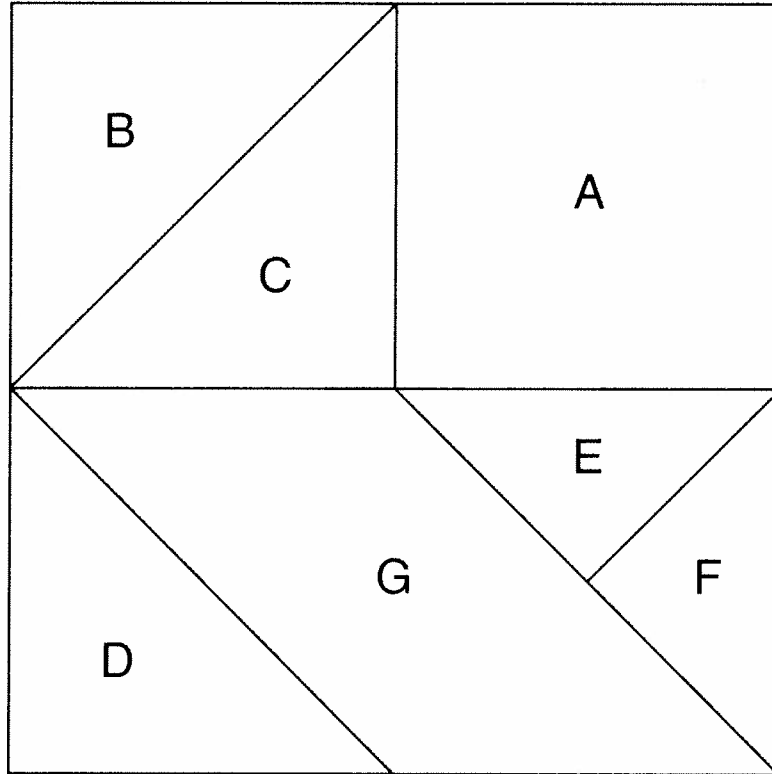
12. $8 - 3.643 =$ _____

LESSON
4•2
Benchmark Fractions


- ◆ Cut out the fraction cards.
- ◆ Remove the benchmark cards 0, $\frac{1}{2}$, and 1. Arrange these benchmark cards in order from left to right, leaving enough space to place fraction cards between them.
- ◆ Sort the remaining 9 cards into 3 piles—fractions closest to 0, fractions closest to $\frac{1}{2}$, and fractions closest to 1.
- ◆ Fill the space between 0, $\frac{1}{2}$, and 1 with your sorted cards, positioning them in order from smallest to largest.
- ◆ Record the order in which you placed the cards on the number line.



$\frac{1}{2}$	$\frac{48}{100}$	$\frac{1}{20}$
$\frac{1}{5}$	$\frac{3}{4}$	$\frac{6}{10}$
$\frac{1}{4}$	$\frac{1}{3}$	$\frac{9}{10}$
0	$\frac{2}{3}$	1

LESSON
4•3
Fractions of a Square

Math Message


1. What fraction of the large square is ...

- a. Square A? _____ b. Triangle B? _____
- c. Triangle E? _____ d. Parallelogram G? _____

2. What fraction of the large square are the following pieces, when put together?
Write a number sentence to show your answer.

- a. Triangles B and C _____
- b. Triangles E and F _____
- c. Square A and Triangle C _____
- d. Square A and Triangle E _____
- e. Triangles E and B _____
- f. Square A and Parallelogram G _____
- g. Triangles D, E, and F and Parallelogram G _____

STUDY LINK
4•3**Adding and Subtracting Fractions**

Add or subtract. Write each answer in simplest form. If possible, rename answers as mixed numbers or whole numbers.

1. $\frac{1}{3} + \frac{1}{6} =$ _____

2. $\frac{3}{4} + \frac{5}{16} =$ _____

3. $\frac{9}{4} + \frac{2}{5} =$ _____

4. $\frac{2}{9} + \frac{4}{9} =$ _____

5. $\frac{1}{6} + \frac{3}{4} =$ _____

6. $\frac{5}{12} + \frac{3}{4} =$ _____

7. $\frac{7}{9} + \frac{2}{5} =$ _____

8. $\frac{5}{4} + \frac{3}{4} =$ _____

9. $\frac{7}{8} - \frac{2}{4} =$ _____

10. $\frac{5}{3} - \frac{2}{5} =$ _____

11. $\frac{11}{12} - \frac{7}{12} =$ _____

12. $\frac{4}{5} - \frac{3}{10} =$ _____

13. $\frac{15}{8} - \frac{3}{24} =$ _____

14. $\frac{3}{5} - \frac{1}{2} =$ _____

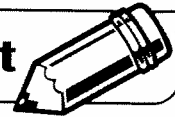
Practice

Solve mentally.

15. $3 - 0.30 =$ _____

16. $0.60 - 0.02 =$ _____

17. $2 - 0.02 =$ _____

LESSON
4•3
Using Fraction Strips to Add and Subtract


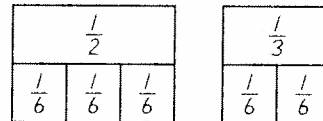
You can use fraction strips to find common denominators, sums, and differences. Study the examples below and then solve the problems.

Example 1: Solve. $\frac{1}{2} + \frac{1}{3} = \frac{\square}{6}$

Step 1 Use fraction strips to model both fractions.



Step 2 Rename each fraction using a common denominator.



Step 3 Write the sum.

$$\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$$

Use your fraction strips to find each of the following sums.

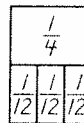
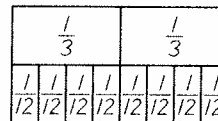
1. $\frac{1}{4} + \frac{2}{3} = \frac{\square}{12}$

2. $\frac{7}{12} + \frac{1}{3} = \frac{\square}{12}$

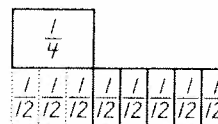
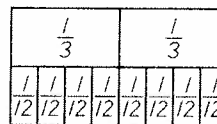
3. $\frac{3}{4} + \frac{1}{8} = \frac{\square}{\square}$

Example 2: Solve. $\frac{2}{3} - \frac{1}{4} = \frac{\square}{12}$

Step 1 Use fraction strips to model both fractions. Rename each fraction using a common denominator.



Step 2 Remove fraction strips to find the difference.



Step 3 Write the difference.

$$\frac{2}{3} - \frac{1}{4} = \frac{5}{12}$$

Use fraction strips to find each of the following differences.

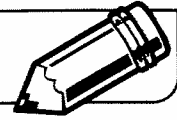
4. $\frac{1}{2} - \frac{1}{3} = \frac{\square}{6}$

5. $\frac{7}{8} - \frac{1}{4} = \frac{\square}{8}$

6. $\frac{1}{3} - \frac{1}{4} = \frac{\square}{\square}$

LESSON
4•3

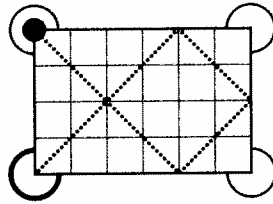
Paper Pool



Paper Pool is played on a rectangular grid. An imaginary ball is hit from the lower left pocket of the rectangular grid at a 45° angle. The ball travels at a 45° angle along the diagonals of the squares making up the grid. When the ball hits a side of the grid, it bounces off that side at a 45° angle and continues to travel along a diagonal that has not already been crossed. Play ends when the ball lands in any of the corner pockets. The length of the ball's path is found by counting the number of diagonals of the individual squares that the ball crosses before landing in a pocket.

3. Ball hits a pocket and play ends.

4. Because the ball crossed the diagonals of 12 squares before landing in a pocket, the length of its path is 12 units.

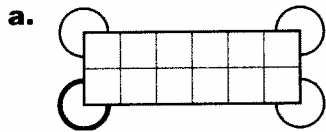


2. Ball hits side(s) and bounces off at a 45° angle.

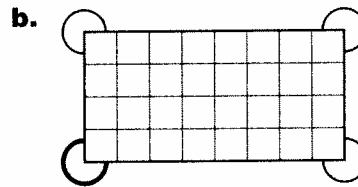
1. Ball is hit from lower left pocket and travels at a 45° angle across the diagonals of the grid squares.

Length of ball's path: 12 units
Dimensions of grid: 4 units by 6 units

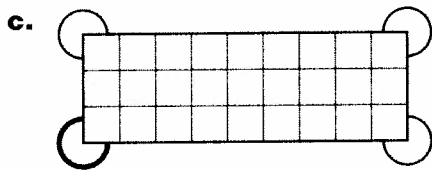
1. Draw the path of the ball on each rectangular grid below. Then record the length of the ball's path.



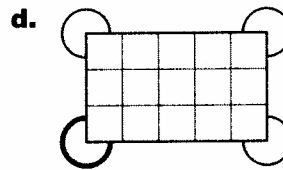
Length of ball's path: _____ units



Length of ball's path: _____ units



Length of ball's path: _____ units



Length of ball's path: _____ units

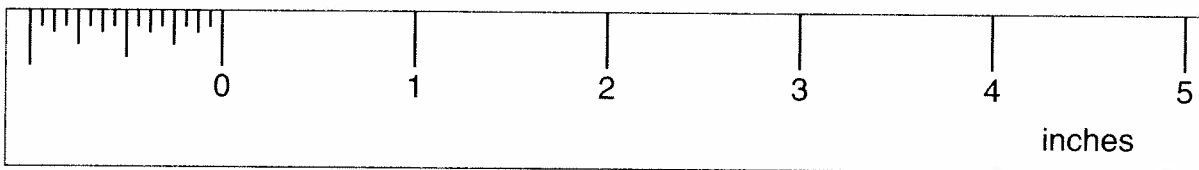
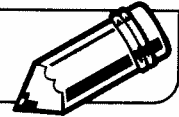
2. For each grid in Problem 1, compare the length of the ball's path to the dimensions of the grid. For example, the length of the ball's path for a 4×6 grid is 12. Describe any patterns you notice.

LESSON
4•4**Math Message**

Cut out the ruler below. Use it to measure line segment AB to the nearest $\frac{1}{16}$ inch.

A _____ B

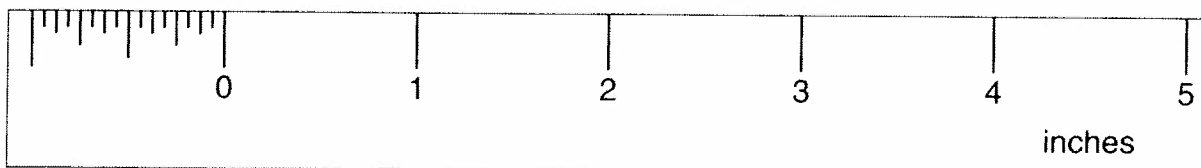
length of \overline{AB} = _____

**LESSON**
4•4**Math Message**

Cut out the ruler below. Use it to measure line segment AB to the nearest $\frac{1}{16}$ inch.

A _____ B

length of \overline{AB} = _____



STUDY LINK
4•4

+ , - Fractions and Mixed Numbers



1. In a national test, eighth-grade students answered the problem shown in the top of the table at the right. Also shown are the 5 possible answers they were given and the percent of students who chose each answer.

- a. What mistake do you think the students who chose C made?

- b. Explain why B is the best estimate.

Estimate the answer to $\frac{12}{13} + \frac{7}{8}$.
 You will not have enough time to solve the problem using paper and pencil.

Possible Answers	Percent Who Chose This Answer
A. 1	7%
B. 2	24%
C. 19	28%
D. 21	27%
E. I don't know.	14%

2. A board is $6\frac{3}{8}$ inches long. Verna wants to cut enough so that it will be $5\frac{1}{8}$ inches long. How much should she cut? _____ (unit)
3. Tim is making papier-mâché. The recipe calls for $1\frac{3}{4}$ cups of paste. Using only $\frac{1}{2}$ -cup, $\frac{1}{4}$ -cup, and $\frac{1}{3}$ -cup measures, how can he measure the correct amount?

Add or subtract. Write your answers as mixed numbers in simplest form. Show your work on the back of the page. Use number sense to check whether each answer is reasonable.

4. $3\frac{1}{4} + 1\frac{1}{4} =$ _____ 5. $4 - 2\frac{1}{4} =$ _____ 6. $1\frac{2}{3} + \frac{2}{3} =$ _____

7. Circle the numbers that are equivalent to $2\frac{3}{4}$.

$1\frac{7}{4}$

$\frac{6}{4}$

$\frac{3}{7}$

$\frac{11}{4}$

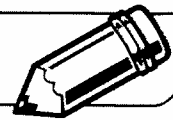
Practice

Solve mentally.

8. $5 * 18 =$ _____ 9. $6 * 41 =$ _____ 10. $9 * 48 =$ _____ 11. $7 * 45 =$ _____

LESSON
4•4

Fraction Counts and Conversions



Most calculators have a function that lets you repeat an operation, such as adding $\frac{1}{4}$ to a number. This is called the constant function. To use the constant function of your calculator to count by $\frac{1}{4}$ s, follow one of the key sequences below, depending on the calculator you are using.

Calculator A	Calculator B
Press: Op1 + 1 n 4 d Op1 Op1 Op1 Op1 Op1 Op1	Press: 1 b/c 4 + + 0 = = = = =
Display: 5 $1\frac{1}{4}$	Display: $\frac{5}{4}$
Press: U$\frac{n}{a} \div \frac{n}{a}$	Press: $\frac{a}{b} \div \frac{c}{d}$
Display: $\frac{5}{4}$	Display: $1\frac{1}{4}$

1. Using a calculator, start at 0 and count by $\frac{1}{4}$ s to answer the following questions.

a. How many counts of $\frac{1}{4}$ are needed to display $\frac{6}{4}$? _____

b. How many counts of $\frac{1}{4}$ are needed to display $1\frac{1}{2}$? _____

2. Use a calculator to convert mixed numbers to improper fractions or whole numbers.

a. $2\frac{3}{4} =$ _____

b. $1\frac{7}{4} =$ _____

c. $2\frac{4}{4} =$ _____

d. $3\frac{12}{4} =$ _____

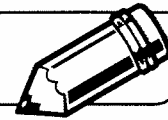
3. How many $\frac{1}{4}$ s are between the following numbers?

a. $\frac{3}{4}$ and 2 _____

b. $\frac{6}{4}$ and $2\frac{3}{4}$ _____

c. $1\frac{3}{4}$ and 4 _____

d. 3 and $4\frac{1}{2}$ _____

LESSON
4•5**Math Message**

Add or subtract. Be ready to explain your solution strategies for Problems 6 and 8.

$$\begin{array}{r} 1. \quad 2\frac{1}{5} \\ + 3\frac{3}{5} \\ \hline \end{array}$$

$$\begin{array}{r} 2. \quad 6\frac{3}{8} \\ + 5\frac{7}{8} \\ \hline \end{array}$$

$$\begin{array}{r} 3. \quad 10\frac{7}{9} \\ - 4\frac{1}{9} \\ \hline \end{array}$$

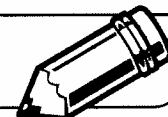
$$\begin{array}{r} 4. \quad 5\frac{1}{4} \\ - 3\frac{3}{4} \\ \hline \end{array}$$

$$\begin{array}{r} 5. \quad 2\frac{2}{8} \\ + 3\frac{1}{8} \\ \hline \end{array}$$

$$\begin{array}{r} 6. \quad 2\frac{1}{4} \\ + 3\frac{1}{8} \\ \hline \end{array}$$

$$\begin{array}{r} 7. \quad 2\frac{7}{12} \\ - 1\frac{6}{12} \\ \hline \end{array}$$

$$\begin{array}{r} 8. \quad 2\frac{7}{12} \\ - 1\frac{1}{2} \\ \hline \end{array}$$

**LESSON**
4•5**Math Message**

Add or subtract. Be ready to explain your solution strategies for Problems 6 and 8.

$$\begin{array}{r} 1. \quad 2\frac{1}{5} \\ + 3\frac{3}{5} \\ \hline \end{array}$$

$$\begin{array}{r} 2. \quad 6\frac{3}{8} \\ + 5\frac{7}{8} \\ \hline \end{array}$$

$$\begin{array}{r} 3. \quad 10\frac{7}{9} \\ - 4\frac{1}{9} \\ \hline \end{array}$$

$$\begin{array}{r} 4. \quad 5\frac{1}{4} \\ - 3\frac{3}{4} \\ \hline \end{array}$$

$$\begin{array}{r} 5. \quad 2\frac{2}{8} \\ + 3\frac{1}{8} \\ \hline \end{array}$$

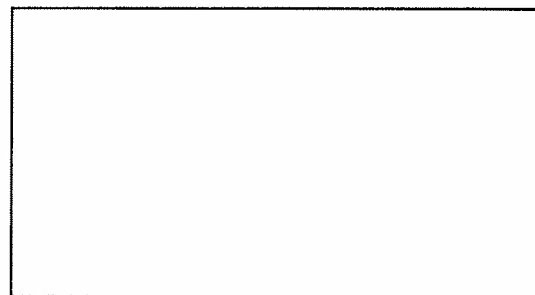
$$\begin{array}{r} 6. \quad 2\frac{1}{4} \\ + 3\frac{1}{8} \\ \hline \end{array}$$

$$\begin{array}{r} 7. \quad 2\frac{7}{12} \\ - 1\frac{6}{12} \\ \hline \end{array}$$

$$\begin{array}{r} 8. \quad 2\frac{7}{12} \\ - 1\frac{1}{2} \\ \hline \end{array}$$

STUDY LINK
4•5
Mixed-Number Practice


1. Answer the following questions about the rectangle shown at the right. Include units in your answers.

 $2\frac{3}{4}$ in.

 $1\frac{1}{2}$ in.

- a. What is the perimeter? _____
- b. If you were to trim this rectangle so that it was a square measuring $1\frac{1}{4}$ inches on a side, how much would you cut from the base? _____ from the height? _____

2. Michael bought 1 peck of Empire apples, 1 peck of Golden Delicious apples, a $\frac{1}{2}$ -bushel of Red Delicious apples, and $1\frac{1}{2}$ bushels of McIntosh apples.

 $1 \text{ peck} = \frac{1}{4} \text{ bushel}$

- a. How many bushels of apples did he buy in all? _____
- b. Michael estimates that he can make about 12 quarts of applesauce per bushel of apples. About how many quarts of applesauce can he make from the apples he bought? _____

Add or subtract. Show your work and estimates on the back of the page.

3. $2\frac{1}{3} + 1\frac{2}{3} =$ _____ 4. $6\frac{1}{3} - 5\frac{2}{3} =$ _____ 5. $4\frac{1}{2} + \frac{2}{3} =$ _____

6. $6 - 5\frac{4}{9} =$ _____ 7. $4\frac{3}{8} - 2\frac{3}{4} =$ _____ 8. $3\frac{1}{4} + 2\frac{3}{4} =$ _____

9. $9 - 2\frac{2}{5} =$ _____ 10. $4\frac{1}{4} - 2\frac{5}{6} =$ _____ 11. $5\frac{1}{4} - 2\frac{7}{10} =$ _____

Practice

Solve mentally.

12. $1\frac{1}{2} + 4\frac{2}{3} + 2\frac{1}{2} + 5\frac{1}{3} =$ _____

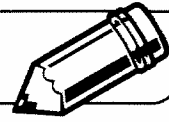
13. $4.5 + 3.4 + 7.5 + 2.5 =$ _____

14. $\$2.35 + \$9.60 + \$8.05 + \$1.99 =$ _____

15. $5\frac{5}{8} + 3\frac{3}{4} + 2\frac{1}{4} + 8\frac{3}{8} =$ _____

LESSON
4•5

Representing Mixed Numbers



Study the example row. Then complete the table. Use Row 5 to represent a mixed number of your choice.

Picture	As Mixed Number	As Sum	Whole Number as Fraction	As Improper Fraction	As Quotient
Example:	$1\frac{1}{3}$	$1 + \frac{1}{3}$	$\frac{3}{3} + \frac{1}{3}$	$\frac{4}{3}$	$4 \div 3$
1.			$\frac{6}{6} + \frac{5}{6}$		$11 \div 6$
2.					
3.					
4.			$\frac{12}{4} + \frac{3}{4}$		$15 \div 4$
5.					

STUDY LINK
4•6

Fraction Multiplication



Use the fraction multiplication algorithm below to solve the following problems.

Fraction Multiplication Algorithm

$$\frac{a}{b} * \frac{c}{d} = \frac{a * c}{b * d}$$

1. $\frac{3}{5} * \frac{2}{4} =$ _____
 2. $\frac{3}{7} * \frac{5}{9} =$ _____
 3. $5 * \frac{3}{8} =$ _____
 4. _____ $= \frac{11}{12} * \frac{1}{4}$
 5. $\frac{5}{6} * \frac{7}{8} =$ _____
 6. $\frac{3}{10} * \frac{7}{10} =$ _____
 7. _____ $= \frac{2}{5} * \frac{7}{9}$
 8. $\frac{4}{7} * 8 =$ _____
 9. $12 * \frac{8}{11} =$ _____
10. South High beat North High in basketball, scoring $\frac{4}{5}$ of the total points. Rachel scored $\frac{1}{4}$ of South High's points. What fraction of the total points did Rachel score? _____
 11. Josh was making raisin muffins for a party. He needed to triple the recipe, which called for $\frac{3}{4}$ cup raisins. How many cups of raisins did he need? _____
 12. At Long Middle School, $\frac{7}{8}$ of the sixth graders live within 1 mile of the school. About $\frac{2}{3}$ of those sixth graders walk to school. None who live a mile or more away walk to school. About what fraction of the sixth graders walk to school? _____
 13. a. For Calista's 12th birthday party, her mom will order pizza. $\frac{3}{4}$ of the girls invited like vegetables on their pizza. However, $\frac{1}{3}$ of those girls won't eat green peppers. What fraction of all the girls will eat a green-pepper-and-onion pizza? _____
 b. If 12 girls are at the party (including Calista), how many girls will not eat a green-pepper-and-onion pizza? _____

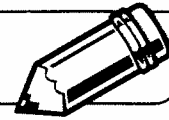
Practice

Solve.

14. $12 * 0.75 =$ _____
15. $0.2 * 0.5 =$ _____
16. $0.4 * 0.25 =$ _____

LESSON
4•6

Modeling Fraction Multiplication



You can use an area model to find a product.

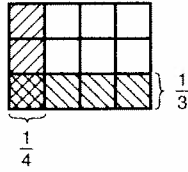
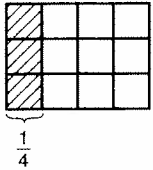
Example: $\frac{1}{4} * \frac{1}{3}$



Shade $\frac{1}{4}$ of
the grid this way:

Shade $\frac{1}{3}$ of
the grid this way:

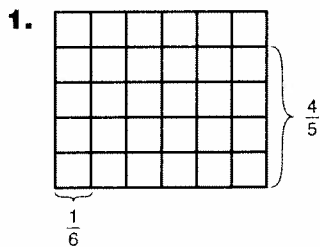
The product is the area
that is double-shaded.



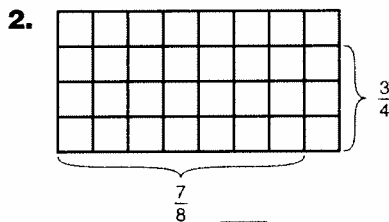
Since $\frac{1}{12}$ of the grid is
double-shaded,

$$\frac{1}{4} * \frac{1}{3} = \frac{1}{12}$$

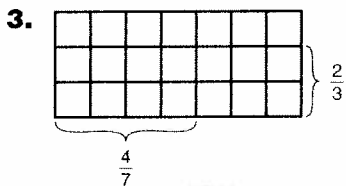
Shade each factor and then find the product.



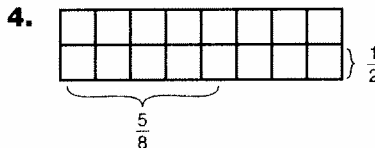
$$\frac{1}{6} * \frac{4}{5} = \frac{\square}{30}$$



$$\frac{3}{4} * \frac{7}{8} = \frac{\square}{\square}$$



$$\frac{2}{3} * \frac{4}{7} = \frac{\square}{21}$$



$$\frac{5}{8} * \frac{1}{2} = \frac{\square}{\square}$$

5. Which of the following represents a general pattern for the special cases in Problems 1–4? Circle the best answer.

A $\frac{a}{b} * \frac{c}{d} = \frac{a * b}{c * d}$

B $\frac{a}{b} * \frac{c}{d} = \frac{a * d}{c * d}$

C $\frac{a}{b} * \frac{c}{d} = \frac{a * c}{b * d}$

D $\frac{a}{b} * \frac{c}{d} = \frac{a + c}{b + d}$