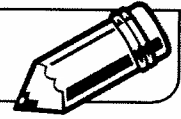


LESSON
6•1**Products and Sums of Reciprocals**

1. Read Statement 1. Then find each reciprocal or product to help you decide whether the statement is true or false.

Statement 1 The product of the reciprocals of two positive numbers is equal to the reciprocal of their product.

- a. The reciprocal of 4 is _____. b. The reciprocal of 6 is _____.
- c. The product of the reciprocals from 1a and 1b is _____.
- d. The product of 4 and 6 is _____.
- e. The reciprocal of the product of 4 and 6 is _____.
- f. Repeat Problems 1a–1e using a different pair of positive numbers.
- g. Do you think Statement 1 is true or false for all positive numbers? Explain.

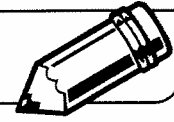
2. Read Statement 2. Then find each reciprocal or sum to help you decide whether the statement is true or false.

Statement 2 The sum of the reciprocals of two positive numbers is equal to the reciprocal of their sum.

- a. The reciprocal of 5 is _____. b. The reciprocal of 10 is _____.
- c. The sum of the reciprocals from 2a and 2b is _____.
- d. The sum of 5 and 10 is _____.
- e. The reciprocal of the sum of 5 and 10 is _____.
- f. Repeat Problems 2a–2e using a different pair of positive numbers.
- g. Do you think Statement 2 is true or false for all numbers having reciprocals? Explain.

LESSON
6•1

Finding Reciprocals



Solve.

1. _____ * 5 = 1

2. _____ * $\frac{1}{2} = 1$

3. _____ * 17 = 1

4. _____ * 0.25 = 1

5. _____ * 0.6 = 1

6. _____ * $n = 1$

7. Explain how you solved Problem 5.

For each number, fill in the circle next to the reciprocal.
 (There may be more than one correct answer.)

8. $\frac{5}{6}$	9. $1\frac{2}{7}$	10. 3	11. 1.25
<input type="checkbox"/> 56	<input type="checkbox"/> $\frac{7}{3}$	<input type="checkbox"/> $\frac{9}{3}$	<input type="checkbox"/> 5.21
<input type="checkbox"/> $1\frac{1}{5}$	<input type="checkbox"/> $\frac{7}{12}$	<input type="checkbox"/> $\frac{3}{9}$	<input type="checkbox"/> $\frac{5}{4}$
<input type="checkbox"/> 1.2	<input type="checkbox"/> $\frac{7}{9}$	<input type="checkbox"/> $\frac{1}{3}$	<input type="checkbox"/> 0.8
<input type="checkbox"/> $\frac{6}{5}$	<input type="checkbox"/> 2.7	<input type="checkbox"/> 1.3	<input type="checkbox"/> $\frac{12}{5}$

12. Explain how you solved Problem 10.

STUDY LINK
6•2

Fraction Division


Division of Fractions Algorithm

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} * \frac{d}{c}$$



Divide. Show your work.

1. $\frac{2}{3} \div \frac{5}{6} =$ _____

2. $1\frac{3}{4} \div \frac{28}{16} =$ _____

3. $\frac{24}{30} \div \frac{4}{5} =$ _____

4. $\frac{7}{3} \div \frac{3}{7} =$ _____

5. $\frac{5}{8} \div \frac{5}{8} =$ _____

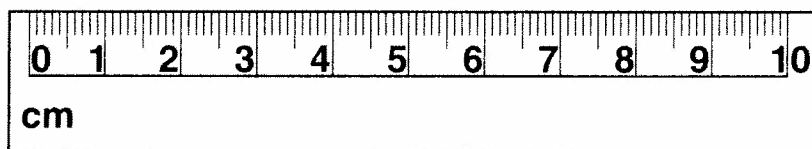
6. $2 \div \frac{1}{4} =$ _____

7. $\frac{1}{7} \div 2\frac{4}{5} =$ _____

8. $5\frac{5}{6} \div 6 =$ _____

Try This

9. How many $\frac{3}{10}$ -centimeter segments are in 3 centimeters? _____ segments
10. How many $\frac{3}{10}$ -centimeter segments are in $4\frac{1}{5}$ centimeters? _____ segments
11. How many $\frac{4}{10}$ -centimeter segments are in $6\frac{4}{5}$ centimeters? _____ segments

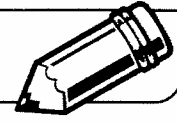

Practice

Round each number to the underlined place.

12. 13.561 _____ 13. 589.3552 _____ 14. 12.9694 _____

LESSON
6•2

Complex Fractions



A *complex fraction* is a fraction whose numerator and/or denominator is also a fraction or a mixed number. Fractions such as

$$\frac{10}{\frac{2}{3}}, \frac{\frac{1}{6}}{\frac{4}{9}}, \text{ and } \frac{22\frac{1}{5}}{\frac{15}{4}}$$

To simplify a complex fraction, rewrite it as a division problem and divide.

Example 1:

Simplify $\frac{10}{\frac{2}{3}}$

$$\begin{aligned} \frac{10}{\frac{2}{3}} &= 10 \div \frac{2}{3} \\ &= 10 * \frac{3}{2} \\ &= \frac{30}{2} \\ &= 15 \end{aligned}$$

Example 2:

Simplify $\frac{\frac{1}{6}}{\frac{4}{9}}$

$$\begin{aligned} \frac{\frac{1}{6}}{\frac{4}{9}} &= \frac{1}{6} \div \frac{4}{9} \\ &= \frac{1}{6} * \frac{9}{4} \\ &= \frac{9}{24} \\ &= \frac{3}{8} \end{aligned}$$

Simplify each complex fraction. Show your work.

1. $\frac{3}{\frac{1}{2}}$

2. $\frac{3}{\frac{7}{6}}$

3. $\frac{\frac{3}{4}}{\frac{5}{6}}$

4. $\frac{6\frac{1}{5}}{\frac{2}{3}}$

Try This

Find each missing divisor.

5. $\frac{1}{4} \div \underline{\hspace{2cm}} = 1\frac{3}{4}$

6. $2\frac{1}{2} \div \underline{\hspace{2cm}} = 1\frac{1}{4}$

LESSON
6•2**Dividing Fractions and Mixed Numbers****Division of Fractions Algorithm**

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} * \frac{d}{c}$$

Divide. Show your work.

1. $\frac{7}{8} \div \frac{3}{6} =$ _____

2. $\frac{11}{15} \div \frac{1}{3} =$ _____

3. $\frac{7}{6} \div \frac{5}{12} =$ _____

4. $6 \div \frac{2}{3} =$ _____

5. $\frac{4}{5} \div 2 =$ _____

6. $\frac{8}{14} \div \frac{8}{14} =$ _____

7. $1\frac{2}{5} \div \frac{3}{10} =$ _____

8. $\frac{16}{3} \div 2\frac{1}{4} =$ _____

9. $2\frac{3}{4} \div \frac{6}{8} =$ _____

Try This

10. $\frac{5}{7} \div 1\frac{3}{5} =$ _____

11. $7 \div 5\frac{1}{3} =$ _____

12. $3\frac{4}{5} \div 8\frac{1}{2} =$ _____

STUDY LINK
6•3

Subtraction of Signed Numbers



For any numbers a and b , $a - b = a + \text{OPP}(b)$, or $a - b = a + (-b)$.

1. Rewrite each subtraction problem as an addition problem. Then solve the problem.

- a. $46 - 19 =$ _____
- b. $-43 - 17 =$ _____
- c. $-5 - (-6.8) =$ _____
- d. $21 - (-21) =$ _____

2. Subtract.

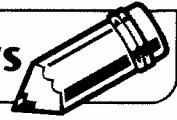
- a. $-72 - (-43) =$ _____
- b. _____ $= 4 - (-39)$
- c. $-\left(\frac{7}{10}\right) - 1\frac{1}{2} =$ _____
- d. $4.8 - (-3.6) =$ _____
- e. _____ $= -2\frac{1}{2} - \frac{3}{4}$
- f. $-\left(\frac{5}{6}\right) - \left(-\frac{1}{3}\right) =$ _____
- g. $-12.3 - 5.9 =$ _____
- h. $-8.5 - (-2.7) =$ _____

3. Fill in the missing numbers.

- a. $19 = 17 -$ _____
- b. $-43 = -26 -$ _____
- c. $\frac{1}{2} -$ _____ $= -1\frac{3}{4}$
- d. _____ $- \left(-2\frac{4}{5}\right) = 3\frac{7}{10}$
- e. $-17.6 =$ _____ $- 13.9$
- f. $83.5 = -62.7 -$ _____
- g. _____ $= 5\frac{3}{4} - 6\frac{3}{16}$
- h. $9.6 -$ _____ $= 10$

Practice

4. $100 = 10^x$; $x =$ _____
5. $10^x = 100$ billion; $x =$ _____
6. 100 million $= 10^x$; $x =$ _____
7. $10^x = 0.00001$; $x =$ _____

LESSON
6•3
Modeling Subtraction with Signed Numbers


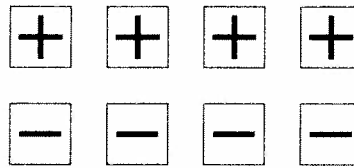
Cut out the + and – tiles on *Math Masters*, page 188.
 Use your tiles to work through each example.

Example 1: $+2 - (-4)$

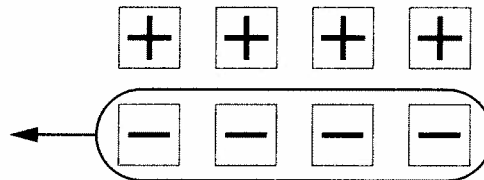
Step 1 Use + tiles to represent 2.



Step 2 Because there are no negative tiles to subtract, add 4 zero pairs.



Step 3 Subtract –4.



Step 4 Count the remaining tiles.

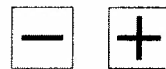
6 (+) tiles are left, so $2 - (-4) = +6$.

Example 2: $-2 - (-3)$

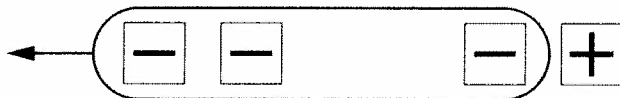
Step 1 Use – tiles to represent –2.



Step 2 Because there are not enough negative tiles to subtract, add 1 more zero pair.



Step 3 Subtract –3.



Step 4 Count the remaining tiles.

1 (+) tile is left, so $-2 - (-3) = +1$.

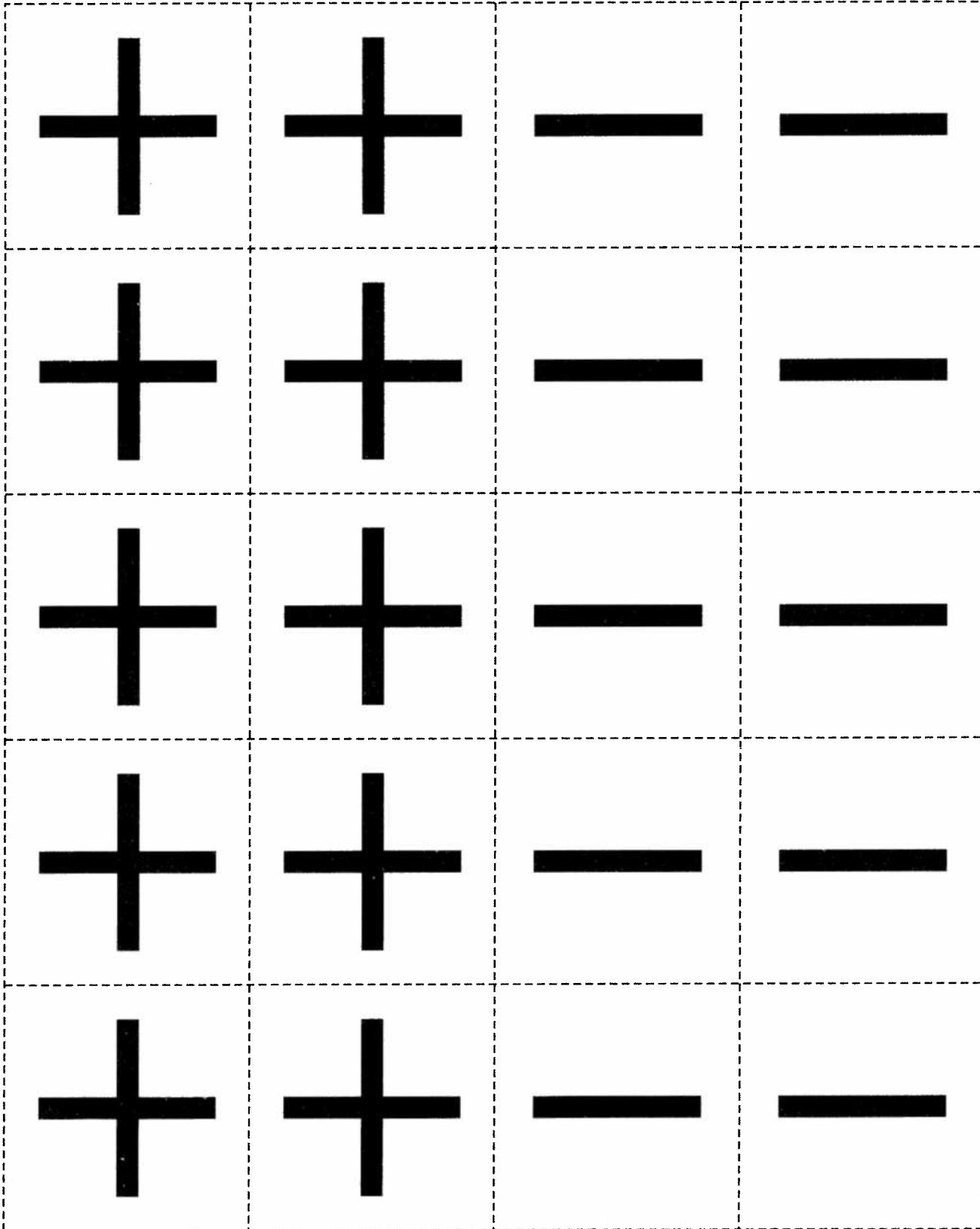
Use your tiles to solve each problem. Record the model you used on the back of this page.

1. $6 - (-2) = \underline{\hspace{2cm}}$

2. $-3 - (+2) = \underline{\hspace{2cm}}$

3. $-7 - (+4) = \underline{\hspace{2cm}}$

4. $-4 - (-5) = \underline{\hspace{2cm}}$

LESSON
6•3**Positive and Negative Tiles**

LESSON
6•3

The Absolute Value of a Number



The absolute value of a number is its distance from 0 on the number line. Use the symbol $| |$ to indicate absolute value. For example, the absolute value of 3 is written $|3|$. On the number line above, both 3 and -3 are 3 units from 0. So, $|3| = 3$ and $|-3| = 3$.

Because absolute value tells the distance and not the direction from 0, the absolute value of any number (except 0) is positive. The absolute value of 0 is 0.

You can use absolute value to find sums of positive and negative numbers.

- ◆ The sum of two positive numbers is the sum of their absolute values.
Example: $3 + 5 = |3| + |5| = 8$.
- ◆ The sum of two negative numbers is the opposite of the sum of their absolute values.
Example: $-3 + -5 = \text{OPP}(|-3| + |-5|) = \text{OPP}(3 + 5) = \text{OPP}(8) = -8$
- ◆ To add two numbers with different signs, first find their absolute values. Then subtract the lesser absolute value from the greater absolute value. Give the result the sign of the number with the greater absolute value.

Example 1: $4 + -7$

$$|4| = 4 \qquad \qquad \qquad |-7| = 7$$

Subtract: $7 - 4 = 3$

Because the negative number has the greater absolute value, the sum is negative. $4 + -7 = -3$

Example 2: $-2 + 8$

$$|-2| = 2 \qquad \qquad \qquad |8| = 8$$

Subtract: $8 - 2 = 6$

Because the positive number has the greater absolute value, the sum is positive. $-2 + 8 = 6$

1. Describe how Example 1 would be different if you found the sum $-4 + 7$.

2. Describe how to find the sum $4 + (-4)$ using absolute values.

STUDY LINK
6•4

*****, **/** of Signed Numbers



A Multiplication Property

- ◆ The product of two numbers with the same sign is positive.
- ◆ The product of two numbers with different signs is negative.

A Division Property

- ◆ The quotient of two numbers with the same sign is positive.
- ◆ The quotient of two numbers with different signs is negative.

Solve.

1. $-12 * 5 =$ _____

2. $-63 / 7 =$ _____

3. $24 \div (-4) =$ _____

4. $-9 * \text{_____} = 54$

5. $-50 / \text{_____} = 10$

6. $-6 * 5 * 8 =$ _____

7. $48 / (-6 - 2) =$ _____

8. $(-8 * 5) + 12 =$ _____

9. $50 * (-23) =$ _____

10. $6 * (12 + 15) =$ _____

11. $(-90 \div 10) + (-45) =$ _____

12. $56 / (-7) / (-4) =$ _____

13. $\text{_____} * (-7) * (-4) = -56$

14. $\text{_____} \div 40 = -9$

Try This

15. $\frac{2}{3} * \left(-\frac{5}{6}\right) =$ _____

16. $(8 * (-3)) - (8 * (-9)) =$ _____

17. $0.25 * (-8) =$ _____

18. $\left(-\frac{3}{4}\right) \div \left(-\frac{1}{2}\right) =$ _____

19. Evaluate each expression for $b = -7$.

a. $(-9 * b) - 27 =$ _____

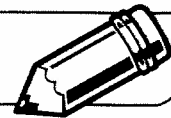
b. $11 * (-b) =$ _____

c. $-b / (-14) =$ _____

d. $b - (b + 16) =$ _____

LESSON
6•4

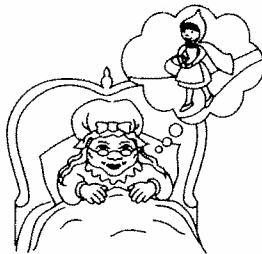
A Multiplication Story



In many fairy tales and children's stories, there are good characters and bad characters. For example, in the story "Little Red Riding Hood," the grandmother is a good character; the wolf is a bad character.

You can use these character situations to remember a multiplication property for positive and negative numbers.

- ◆ When something good (+) happens to a good (+) character, we think it is good (+).



- ◆ When something bad (-) happens to a good (+) character, we think it is bad (-).



- ◆ When something good (+) happens to a bad (-) character, we think it is bad (-).



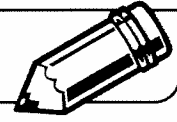
- ◆ When something bad (-) happens to a bad (-) character, we think it is good (+).



Example: Solve $-4 * 5 = ?$ Think, "When something bad (-4) happens to a good (+5) character, we think it is bad (-)." So, $-4 * 5 = -20$.

LESSON
6•4

Patterns with Signed Numbers



1. Multiply each number in the far left column by each number in the top row. Look for patterns. Use your calculator as few times as possible to complete the table.

*	-11	111	-1,111
-11			
111			
-1,111			
11,111			

2. Use the patterns from the table above to predict the products below. Then check each prediction with your calculator.

	*	-11	111
Prediction	111,111		
Actual	111,111		

3. Divide each number in the far left column by each number in the top row. Look for patterns. Use your calculator as few times as possible to complete the table. Write your own number pattern in last row.

	Divisor		
/	-99	999	-9,999
-12			
34			
-45			
67			

STUDY LINK
6•5

Turn-Around Patterns



Fill in the missing numbers in the tables. Look for patterns in the results.

1.

x	y	OPP (x)	OPP (y)	$x + y$	$y + x$	$x - y$	$y - x$
7	9	-7	-9	-16			
-2	12						
-3	-9						
$\frac{2}{3}$	$\frac{5}{6}$						
2.7	-1.9						
2^2	2^3						

Which patterns did you find in your completed table?

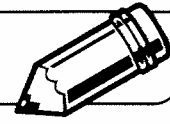
2.

x	y	$\frac{1}{x}$	$\frac{1}{y}$	$x * y$	$y * x$	$x \div y$	$y \div x$
7	9	$\frac{1}{7}$	$\frac{1}{9}$	63			
-2	12						
-3	-9						
$\frac{2}{3}$	$\frac{5}{6}$						
2.7	-1.9						
2^2	2^3						

Which patterns did you find in your completed table?

LESSON
6•5

Properties of Numbers



For each statement below, indicate whether it is always true or can be false. If the statement can be false, give an example.

True or false?

Example

1. $\frac{a}{b} + \frac{c}{d} = \frac{a+c}{b+d}$

2. $\frac{a}{b} * \frac{c}{d} = \frac{a*c}{b*d}$

3. $\frac{a}{b} - \frac{c}{d} = \frac{a-c}{b-d}$

4. $\frac{a}{b} \div \frac{c}{d} = \frac{a \div c}{b \div d}$

5. Explain why giving only one example for a true statement is not enough to prove that it is true.

Try This

6. Correct each false statement in Problems 1–4 so the statement is true for all special cases. Give one example for each statement.

LESSON
6•5

Renaming Repeating Decimals



You can use a power-of-10 strategy when renaming a repeating decimal as a fraction. Work through each of the examples shown below.

Example 1: Rename $0.\overline{3}$ as a fraction.

Let $1x = 0.3333\dots$

Because one digit repeats, multiply both sides by 10 to eliminate the repeating digits to the right of the decimal point.

Subtract

Divide to solve for x .

Simplify.

$0.\overline{3}$ renamed as a fraction is $\frac{1}{3}$.

If $1x = 0.333\dots$,
then $10x = 3.33\dots$

$$\begin{array}{r} 10x = 3.333 \\ - 1x = 0.333 \\ \hline 9x = 3 \end{array}$$

$$\frac{9x}{9} = \frac{3}{9}$$

$$x = \frac{3}{9} = \frac{1}{3}$$

Example 2: Rename $0.\overline{45}$ as a fraction.

Let $1x = 0.4545\dots$

Because two digits repeat, multiply both sides by 100 to eliminate the repeating digits to the right of the decimal point.

Subtract.

Divide to solve for x .

Simplify.

$0.\overline{45}$ renamed as a fraction is $\frac{5}{11}$.

If $1x = 0.454545\dots$,
then $100x = 45.45\dots$

$$\begin{array}{r} 100x = 45.4545 \\ - 1x = 0.4545 \\ \hline 99x = 45 \end{array}$$

$$\frac{99x}{99} = \frac{45}{99}$$

$$x = \frac{45}{99} = \frac{5}{11}$$

Rename each repeating decimal as a fraction.

1. $0.\overline{7} =$ _____

2. $0.\overline{25} =$ _____

3. Compare the denominators in the examples to the denominators of your answers for Problems 1 and 2. Use any patterns you notice to mentally rename $0.\overline{5}$ and $0.\overline{32}$. Check your answers with a calculator.

a. $0.\overline{5} =$ _____

b. $0.\overline{32} =$ _____

STUDY LINK
6•6

Using Order of Operations



Please Excuse My Dear Aunt Sally

Parentheses Exponents Multiplication Division Addition Subtraction

Evaluate each expression.

1. $5 + 6 * 3 - 2 =$ _____

2. $4 * 9 / 2 + (-4 + 6) =$ _____

3. $\frac{1}{2} + \frac{5}{8} * \frac{1}{2} \div 2 =$ _____

4. $(2.3 + 7.8) * 4 + 3 =$ _____

5. $4^2 + 7(3 - (-5)) =$ _____

6. $((2 * 4) + 3) * 6 / 2 =$ _____

Evaluate the following expressions for $m = -3$.

7. $-\frac{m}{m} + 6 - 4 =$ _____

8. $((4 + 11) * -3) / 9 * (-m) =$ _____

9. $m^2 + (-m^3) - 8 =$ _____

10. $\frac{1}{2} * m \div \frac{5}{4} + \frac{3}{5} - \frac{1}{10} =$ _____

Practice

Find each missing number.

11. 3 gal 7 qt = 4 gal _____ qt

12. 5 gal 3 qt = _____ qt

13. 13 pt = _____ qt _____ pt

14. 10 c = _____ qt _____ pt

15. 18 qt = _____ gal _____ pt

Units of Capacity

2 cups (c) = 1 pint (pt)

2 pints = 1 quart (qt)

4 quarts = 1 gallon (gal)