

STUDY LINK
9•1

Multiplying Sums



1. For each expression in the top row, find one or more equivalent expressions below it. Fill in the oval next to each equivalent expression.

a. $(8 + 7) * 4$

$(8 * 4) + (7 * 4)$

$4 * (7 + 8)$

$(8 + 4) * 7$

$(8 + 4) * (7 + 4)$

b. $(6 * 5) + (6 * 8)$

$(8 * 6) + (5 * 6)$

$6 * (5 + 8)$

$(8 + 5) * 6$

$(6 + 5) * (6 + 8)$

c. $3 * (9 + 4)$

$(9 + 4) * (3 + 4)$

$9 * (3 + 4)$

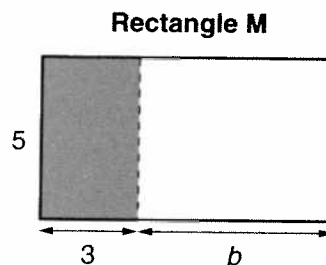
$(4 + 9) * 3$

$(9 * 3) + (4 * 3)$

2. The area of Rectangle M is 45 square units.

a. What is the value of b ? _____

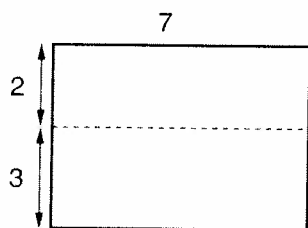
b. Write 2 different number sentences to describe the area of the unshaded part of Rectangle M.



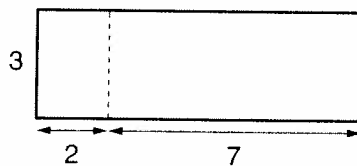
$(\text{---} - \text{---}) * \text{---} = \text{---}$ $(\text{---} * \text{---}) - (\text{---} * \text{---}) = \text{---}$

3. Each of the following expressions describes the area of one of the rectangles below. Write the letter of the rectangle next to its expression.

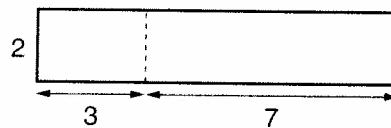
Rectangle N



Rectangle O



Rectangle P



a. $(3 + 2) * 7$ _____

b. $(2 * 3) + (7 * 3)$ _____

c. $(7 + 2) * 3$ _____

d. $(3 * 7) + (2 * 7)$ _____

e. $2 * (7 + 3)$ _____

f. $3 * (2 + 7)$ _____

4. Sandra wants to buy envelopes and stamps to send cards to 8 friends. Envelopes cost \$0.10 and stamps cost \$0.39. How much will she spend? _____

Write a number model to show how you solved the problem.

STUDY LINK
9·2

Using the Distributive Property



Reminder: $a * (x + y) = (a * x) + (a * y)$
 $a * (x - y) = (a * x) - (a * y)$

1. Use the distributive property to rewrite each expression.

a. $7 * (3 + 4) = (\text{_____} * \text{_____}) + (\text{_____} * \text{_____})$

b. $7 * (3 + \pi) = (\text{_____} * \text{_____}) + (\text{_____} * \text{_____})$

c. $7 * (3 + y) = (\text{_____} * \text{_____}) + (\text{_____} * \text{_____})$

d. $7 * (3 + (2 * 4)) = (\text{_____} * \text{_____}) + (\text{_____} * (2 * 4))$

e. $7 * (3 + (2 * \pi)) = (\text{_____} * \text{_____}) + (\text{_____} * (2 * \text{_____}))$

f. $7 * (3 + (2 * y)) = (\text{_____} * \text{_____}) + (\text{_____} * (\text{_____} * \text{_____}))$

2. Use the distributive property to solve each problem. Study the first one.

a. $7 * (110 + 25) = \underline{(7 * 110) + (7 * 25) = 770 + 175 = 945}$

b. $20 * (42 - 19) = \underline{\hspace{10em}}$

c. $(32 + 50) * 40 = \underline{\hspace{10em}}$

d. $(90 - 8) * 11 = \underline{\hspace{10em}}$

e. $9 * (15 + 25) = \underline{\hspace{10em}}$

3. Circle the statements that are examples of the distributive property.

a. $(80 * 5) + (120 * 5) = (80 + 120) * 5$ b. $6 * (3 - 0.5) = (6 * 3) - 0.5$

c. $12(d - t) = 12d - 12t$ d. $(a + c) * n = a * n + c * n$

e. $(16 + 4m) + 9.7 = 16 + (4m + 9.7)$ f. $(9 * \frac{1}{2}) - (\frac{1}{3} * \frac{1}{2}) = (9 - \frac{1}{3}) * \frac{1}{2}$

Practice

Write each quotient in lowest terms.

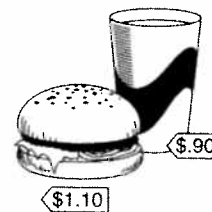
4. $\frac{1}{5} \div \frac{1}{15} = \underline{\hspace{2em}}$

5. $\frac{3}{7} \div \frac{6}{11} = \underline{\hspace{2em}}$

6. $1\frac{1}{19} \div 7\frac{1}{2} = \underline{\hspace{2em}}$

LESSON
9•2**Applying the Distributive Property**

1. Cheng and 5 of his friends are buying lunch. Each person gets a hamburger and a soda. How much money will they spend in all?



Write a number model to show how you solved the problem. _____

Answer _____

Explain how the distributive property can help you solve Problem 1.

2. Minowa signed her new book at a local bookstore. In the morning she signed 36 books, and in the afternoon she signed 51 books. It took her 5 minutes to sign each. How much time did she spend signing books?

Write a number model to show how you solved the problem. _____

Answer _____

3. Ms. Hays bought fabric for the school musical chorus. She bought 4 yards each of one kind for 30 group costumes and 4 yards each of another kind for 6 soloists. How many yards did she buy in all?

Write a number model to show how you solved the problem. _____

Answer _____

4. Mr. Katz gave a party because all the students got 100% on their math test. He had budgeted \$1.15 per student. It turned out that he saved \$0.25 per student. If there are 30 students, how much did he spend?

Write a number model to show how you solved the problem. _____

Answer _____

Fill in the missing numbers according to the distributive property.

5. $28 * 6 = (\underline{\quad} + \underline{\quad}) * 6$ 6. $(\underline{\quad} * 6) + (\underline{\quad} * 6) = (20 + 8) * 6$

STUDY LINK
9·3

Combining Like Terms



Simplify each expression by rewriting it as a single term.

1. $3x + 12x =$ _____

2. $(1\frac{3}{5})y - (1\frac{3}{10})y =$ _____

3. $-(5t) - 6t =$ _____

4. $4d + (-3d) =$ _____

Complete each equation.

5. $15k = (9 - \text{_____})k$

6. $3.6p - p = \text{_____} - 0.4p$

7. $(8 + \text{_____}) * m = 5m$

8. $\text{_____}j - 4.5j = 3.8j$

Simplify each expression by combining like terms. Check your answers by substituting the given values for the variables. Show your work on the back of this sheet.

Example: $18 + 6m + 2m + 26$

Combine the m terms. $6m + 2m = 8m$

Combine the number, or constant, terms. $18 + 26 = 44$

So, $18 + 6m + 2m + 26 = 8m + 44$.

Check: Substitute 5 for m .

$$18 + (6 * 5) + (2 * 5) + 26 = (8 * 5) + 44$$

$$18 + 30 + 10 + 26 = 40 + 44$$

$$84 = 84$$

9. $8b + 9 + 4b - 3b + (-2b) - (-5) =$ _____

Check for: $b = -6$

10. $\frac{1}{2}a + \frac{3}{4}t + \frac{2}{3}a + (-\frac{1}{2}t) =$ _____

Check for: $a = 2$ and $t = -2$

Practice

11. $-117 + 64 =$ _____

12. $-9 - (-32) =$ _____

13. $-12 * (-11) =$ _____

14. $\frac{57}{-3} =$ _____

LESSON
9·3**Simplifying and Evaluating Expressions**

Each expression on the left of the equal sign can be simplified to the expression on the right. Fill in the missing variable or constant terms.

1. $9x + \underline{\hspace{2cm}} + \underline{\hspace{2cm}} + 10 = 7x + 13$

2. $4m - 8n + \underline{\hspace{2cm}} + \underline{\hspace{2cm}} - 6 + \underline{\hspace{2cm}} = -4m + 32n + 6$

3. $-2t - (-2v) + 2w + \underline{\hspace{2cm}} + \underline{\hspace{2cm}} - \underline{\hspace{2cm}} = -4t - 2v + 4w$

4. $3c + 2c + 6d - 4d + 10 - \underline{\hspace{2cm}} + \underline{\hspace{2cm}} = 2d + 2$

5. $8f - \underline{\hspace{2cm}} + 4g - \underline{\hspace{2cm}} + 13 + \underline{\hspace{2cm}} = -f + g + 10$

Simplify each expression below. Then evaluate the expression for $x = -2$, $y = 3$, and $z = -4$.

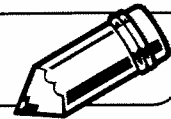
6. $2y - 6z + 4x - 2z - 8y - 12x$ _____

7. $3x - 10 - 4x - 9y + 6x - 4z$ _____

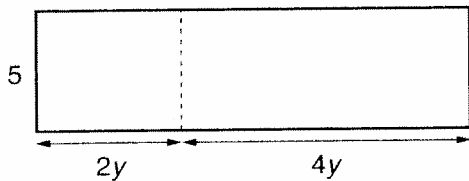
8. $2x + 9 - 6z + 3y - 6z - 15$ _____

9. $3z - 5x + 9y + x - y + 5z$ _____

10. $\frac{1}{2}x - \frac{2}{3}y + \frac{3}{4}z - \frac{3}{2}x$ _____

LESSON
9•4
Area and Variables

Math Message

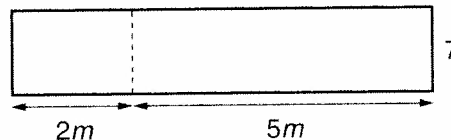
Write an expression to show how you could find the area of each rectangle.
 Then find the area by substituting the given value for the variable.

Rectangle A


Expression _____

 Evaluate for $y = 3$. _____

If $y = 3$, then the area of
 Rectangle A is _____ units².

Rectangle B


Expression _____

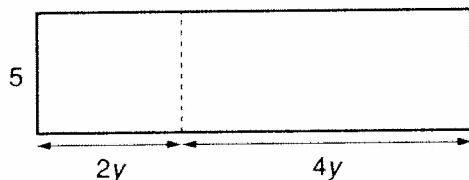
 Evaluate for $m = 5$. _____

If $m = 5$, then the area of
 Rectangle B is _____ units².

LESSON
9•4
Area and Variables

Math Message

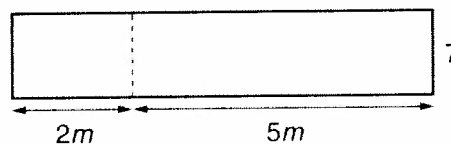
Write an expression to show how you could find the area of each rectangle.
 Then find the area by substituting the given value for the variable.

Rectangle A


Expression _____

 Evaluate for $y = 3$. _____

If $y = 3$, then the area of
 Rectangle A is _____ units².

Rectangle B


Expression _____

 Evaluate for $m = 5$. _____

If $m = 5$, then the area of
 Rectangle B is _____ units².

STUDY LINK
9·4**Simplifying Expressions**

Simplify each expression by removing parentheses and combining like terms. Check by substituting the given values for the variables. Show your work.

1. $7(7 + 5f) + (f + 6)10$ _____

Check: Substitute $-\frac{1}{5}$ for f .

2. $3(4 + 5m) - 12 + (-3m)$ _____

Check: Substitute $\frac{1}{3}$ for m .

3. $(12 - 3 + 5k)6 + 4k - 2(k + 5)$ _____

Check: Substitute 0.5 for k .

4. $5(y - b) + 3b - 6y + 4(6 + b)$ _____

Check: Substitute 1 for y and $\frac{2}{3}$ for b .

Practice

Find each product or quotient.

5. $0.658 * 10^5$ _____

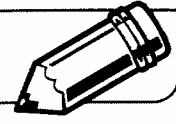
6. $234.8 \div 10^3$ _____

7. $5,163 * 10^{-4}$ _____

8. $7.96 \div 10^2$ _____

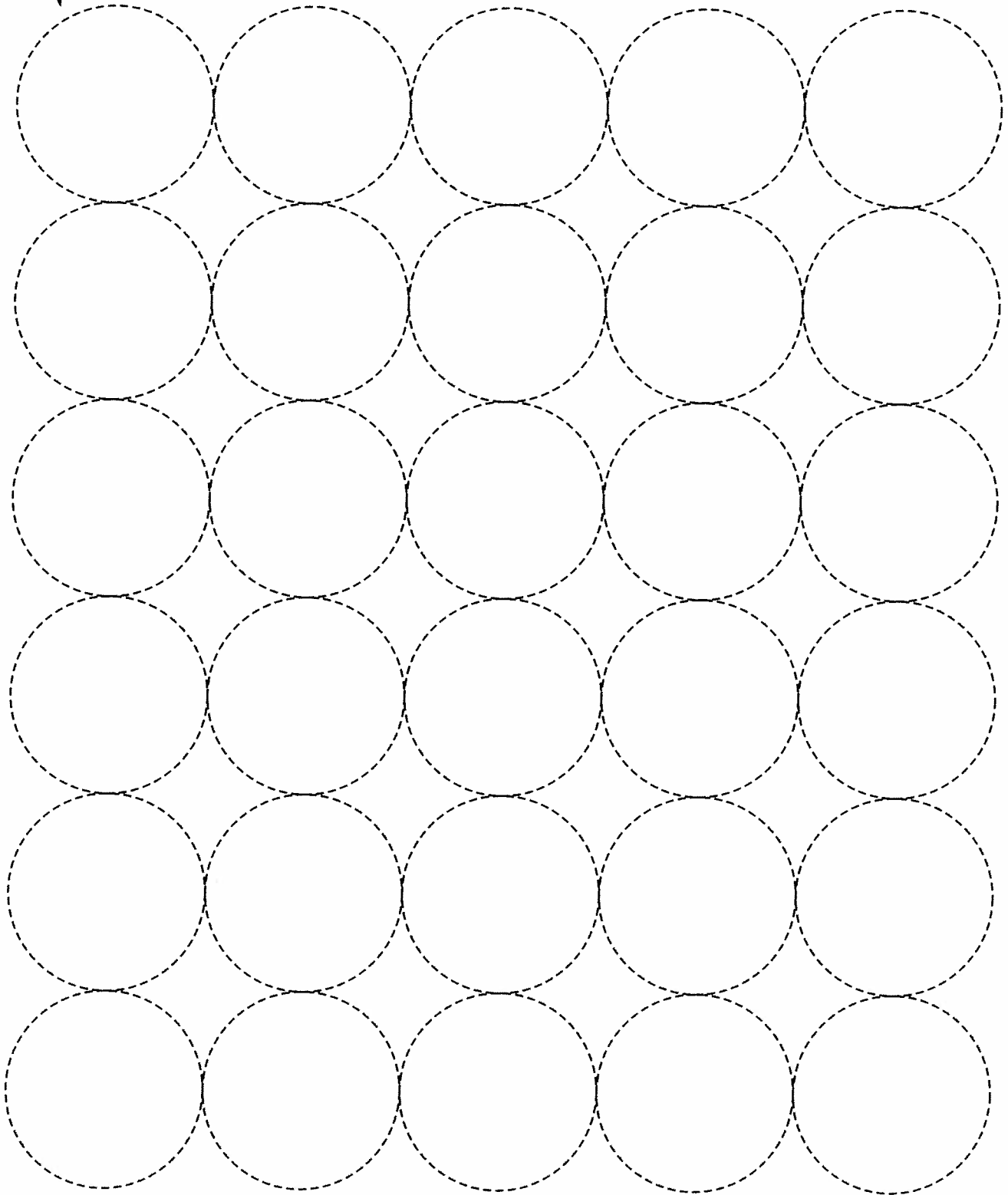
LESSON
9•4

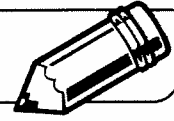
Variable-Term Tiles



LESSON
9•4

Constant-Term Tiles

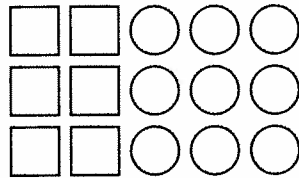


LESSON
9•4**Modeling and Simplifying Algebraic Expressions**

For each expression, use the \square tiles to represent the variable terms and the \bigcirc tiles to represent the constant terms.

Example: Simplify $3(2x + 3)$.

Step 1 $3(2x + 3)$ means 3 of $(2x + 3)$. Model $(2x + 3)$ three times.



Step 2 Combine like terms.

Combine variable terms.



Combine constant terms.



Step 3 Simplify.

$$3(2x + 3) = 6x + 9$$

Use tiles or pictures to model each expression. Combine like terms and simplify.

1. $2(y + 4) =$ _____

2. $4(2k + 3) =$ _____

3. $5(w + 2) + 5 =$ _____

4. $6(1 + x) + 3x =$ _____

5. $p + 2(p + 3) + 2p =$ _____

6. $2(n + 3n) + n + 1 =$ _____

STUDY LINK
9•5

Equivalent Equations



Each equation in Column 2 is equivalent to an equation in Column 1. Solve each equation in Column 1. Write *Any number* if all numbers are solutions of the equation.

Match each equation in Column 1 with an equivalent equation in Column 2. Write the letter label of the equation in Column 1 next to the equivalent equation in Column 2.

Column 1

A $4x - 2 = 6$

Solution _____

B $3s = -6$

Solution _____

C $3y - 2y = y$

Solution _____

D $5a = 7a$

Solution _____

Column 2

_____ $6j + 8 = 8 + 6j$

_____ $2c - 1 = 3$

_____ $6w = -12$

_____ $\frac{2h}{2h} = 1$

_____ $\frac{3q}{3} - 6 = -4$

A _____ $3(r + 4) = 18$

_____ $2(5x + 1) = 10x + 2$

_____ $-5x - 5(2 - x) = 2(x - 7)$

_____ $s = 0$

_____ $5b - 3 - 2b = 6b + 3$

_____ $\frac{t}{4} + 3 = 2\frac{1}{2}$

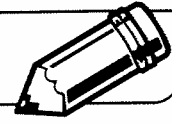
_____ $6z = 12$

_____ $2a = (4 + 7)a$

Practice

Write each product or quotient in exponential notation.

1. $2^2 * 2^3$ _____ 2. $\frac{10^4}{10^2}$ _____ 3. $5^2 * 5^2$ _____ 4. $\frac{4^3}{4^2}$ _____

LESSON
9•5**Revisiting Pan Balances**

Solve the equations. For each step, record the operation you use and the equation that results.

Check your solution by substituting it for the variable in the original equation.

1. Original equation _____ = _____

Operation _____

_____ = _____
 _____ = _____
 _____ = _____

2. Original equation _____ = _____

Operation _____

_____ = _____
 _____ = _____
 _____ = _____

3. Original equation _____ = _____

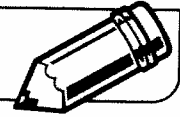
Operation _____

_____ = _____
 _____ = _____
 _____ = _____

4. Original equation _____ = _____

Operation _____

_____ = _____
 _____ = _____
 _____ = _____

LESSON
9•5**Writing and Solving Equations**

Sometimes you need to translate words into algebraic expressions to solve problems.

Example: The second of two numbers is 4 times the first. Their sum is 50.
Find the numbers.

If $n =$ the first number, then

$4n =$ the second number, and $n + 4n = 50$.

Because $5n = 50$, $n = 10$.

The first number is 10 and the second number is $4(10)$, or 40.

For each problem, translate the words into algebraic expressions. Then write an equation and solve it.

1. The larger of two numbers is 12 more than the smaller. Their sum is 84.
Find the numbers.

Equation _____

Smaller number _____

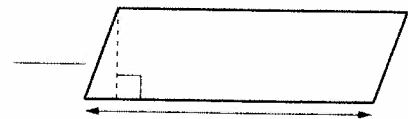
Larger number _____

2. Mr. Zock's sixth-grade class of 29 students has 9 more boys than girls.
How many girls are in the class?

Equation _____ Number of girls _____

Sometimes it helps to label a diagram when you are translating words into algebraic expressions.

3. The base (b) of a parallelogram is 3 times as long as an adjacent side (s). The perimeter of the parallelogram is 64 m. What is the length of the base?



$b =$ _____

Label the diagram at the right. Then write an equation and solve it.

Equation _____

Length of the base _____ units

LESSON
9•5**More Simplifying and Solving of Equations**

Simplify each equation. Then solve it. Show your work.

1. $4(5t - 7) = 10t + 2$

Solution _____

2. $18(m + 6) = 15m - 6$

Solution _____

3. $4(12 - 8w) = w - 18$

Solution _____

4. $3g + 8(2g - 6) = 2 + 14g$

Solution _____

5. $-7(1 - 4y) = 13(2y - 3)$

Solution _____

6. $4n + 5(7n - 3) = 9(n - 5)$

Solution _____

7. $2(6v + 3) = 18 - 3(16 - 3v)$

Solution _____

8. $-5 + (-15d + 1) = 2(7d - 16) - d$

Solution _____

STUDY LINK
9•6

Expressions and Equations



Solve.

1. $3x + 9 = 30$ $x =$ _____ 2. $73 = \frac{1}{2}(108 + f)$ $f =$ _____

3. $55 = (9 - d) * 11$ $d =$ _____ 4. $(m * 15) + (m * 6) = 42$ $m =$ _____

Simplify these expressions by combining like terms.

5. $8y + 27 + 6y + (-4)$ _____ 6. $7b + 17 - 9b + 15$ _____

7. $3f - 80 + 25 - 10k$ _____ 8. $240 + 5g + 3(10g - 5)$ _____

Circle all expressions that are equivalent to the original. There may be more than one. Check your answer by substituting values for the variable.

9. Original: $3r + 17 - 2r + 6$

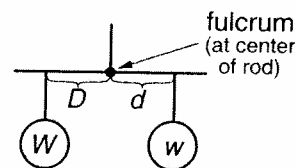
$5r + 23$ $23 - r$ $r + 23$ $13 + r$

10. Original: $8(9 + b) - 4b$

$89 - 3b$ $72 - 3b$ $4b + 72$ $72 - (-4b)$

Try This

11. The top mobile is in balance. The fulcrum is at the center of the rod. A mobile will balance when
- $W * D = w * d$
- .



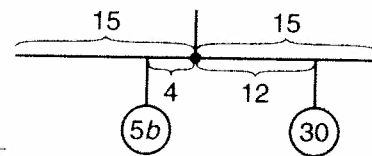
Look at the bottom mobile. What is the weight of the object on the left?

Write and solve an equation to answer the question.

$W =$ _____ $D =$ _____ $w =$ _____ $d =$ _____

Equation _____ Solution _____

The weight of the object on the left is _____ units.


Practice

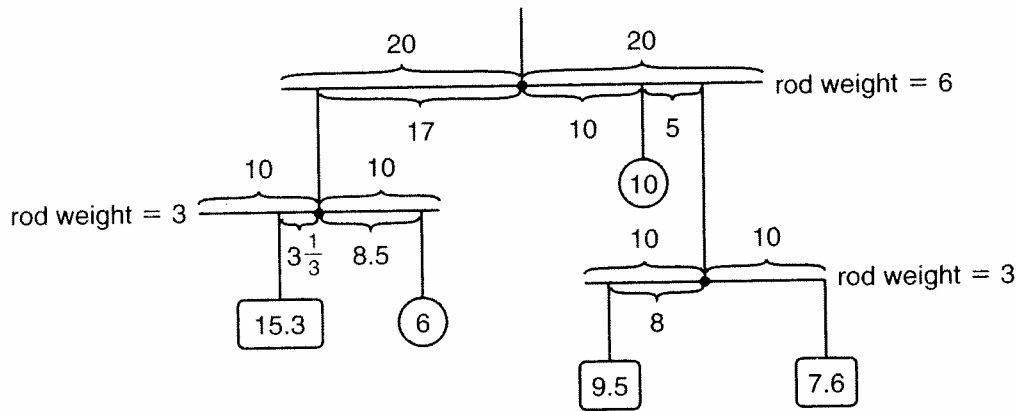
12. $8\frac{1}{3} - 2\frac{7}{8}$ _____

13. $3\frac{5}{6} * 24$ _____

14. $25 \div 4\frac{3}{8}$ _____

LESSON
9•6
Challenge: Balancing a Mobile


In the mobile shown below, each rod is suspended at its center.

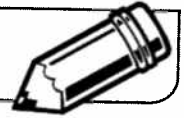


1. Is each rod in perfect balance? _____

If not, which of the rods is not balanced? _____

Explain how you found the answer. _____

2. If you found a rod in the mobile that was not balanced, how would you move exactly one of the suspending wires so the rod would balance?

LESSON
9·7**Evaluating Expressions**

Name _____

Math MessageEvaluate the expression $(b^2 * 4) + k$ for the following values:

- $b = 3$ and $k = 5$ _____
- $b = \frac{1}{2}$ and $k = \frac{3}{4}$ _____
- $b = -2$ and $k = -10$ _____
- $b = 5$ and $k = 115$ _____

Name _____

Math MessageEvaluate the expression $(b^2 * 4) + k$ for the following values:

- $b = 3$ and $k = 5$ _____
- $b = \frac{1}{2}$ and $k = \frac{3}{4}$ _____
- $b = -2$ and $k = -10$ _____
- $b = 5$ and $k = 115$ _____

Name _____

Math MessageEvaluate the expression $(b^2 * 4) + k$ for the following values:

- $b = 3$ and $k = 5$ _____
- $b = \frac{1}{2}$ and $k = \frac{3}{4}$ _____
- $b = -2$ and $k = -10$ _____
- $b = 5$ and $k = 115$ _____

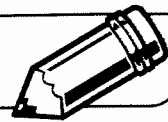
Name _____

Math MessageEvaluate the expression $(b^2 * 4) + k$ for the following values:

- $b = 3$ and $k = 5$ _____
- $b = \frac{1}{2}$ and $k = \frac{3}{4}$ _____
- $b = -2$ and $k = -10$ _____
- $b = 5$ and $k = 115$ _____

LESSON
9•7

Sample Spreadsheet



address box cancel accept display bar

	A	B	C	D
	Supplies (\$)			
	D3	<input checked="" type="checkbox"/> $= B3 * C3$		
1	item name	unit price	quantity	totals
2				
3	pencils	0.29	6	1.74
4	graph paper	1.19	2	2.38
5	ruler	0.50	1	0.50
6	book	5.95	1	5.95
7				
8	Subtotal			10.57
9	tax 7%			0.74
10	Total			\$11.31

STUDY LINK
9·7

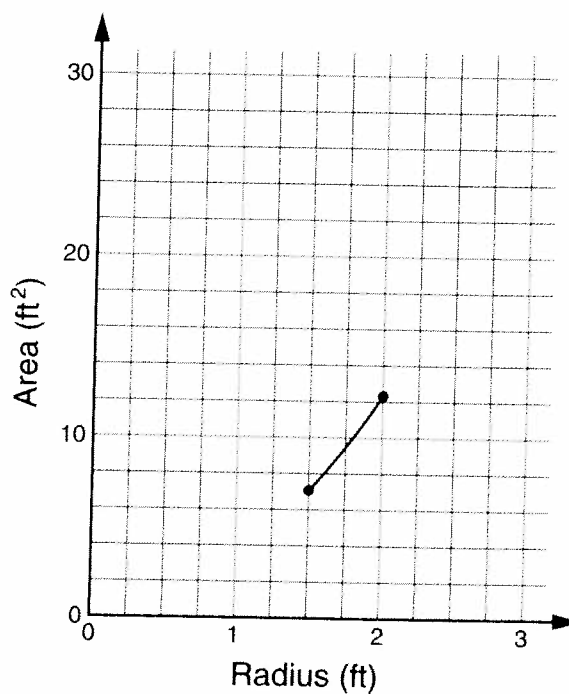
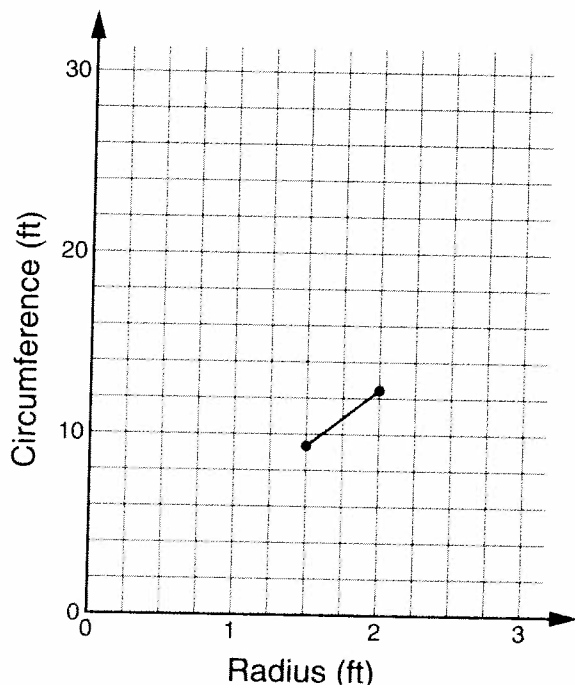
Circumferences and Areas of Circles



Circles			
	A	B	C
1	circumferences and areas of circles		
2	radius (ft)	circumference (ft)	area (ft ²)
3	r	$2\pi r$	πr^2
4	0.5		
5	1.0		
6	1.5	9.4	7.1
7	2.0	12.6	12.6
8	2.5		
9	3.0		

1. Complete the spreadsheet at the left. For each radius, calculate the circumference and area of a circle having that radius. Round your answers to tenths.

2. Use the data in the spreadsheet to graph the number pairs for radius and circumference on the first grid below. Then graph the number pairs for radius and area on the second grid below. Connect the plotted points.



3. A circular tabletop has an area of 23 square feet. Use the second line graph to estimate the radius of the tabletop. Radius: About _____ (unit)

LESSON
9·7

Stopping Distance for an Automobile



Drivers sometimes need to stop quickly. The time it takes to stop depends on the car's speed. A driver takes about $\frac{3}{4}$ second to react before actually stepping on the brake pedal. After the brake has been depressed, additional time passes before the car comes to a complete stop.

The spreadsheet below shows the minimum stopping distances for various speeds.

- The spreadsheet is not completely filled in. Calculate and record the numbers for the cells in rows 9, 10, and 11. (*Hint: Use the formulas in cells B4, C4, and D4.*)

Stopping Distances				
	A	B	C	D
1	minimum stopping distance on a dry, level, concrete surface			
2				
3	speed (mph)	reaction-time distance (ft)	braking distance (ft)	total stopping distance (ft)
4		distance = 1.1 * speed	distance = 0.06 * speed ²	distance = 1.1 * speed + 0.06 * speed ²
5	10	11	6	17
6	20	22	24	46
7	30	33	54	87
8	40	44	96	140
9	50			
10	60			
11	70			

- Circle any cell that contains labels. D4 B10 C6 A3
- Circle any cell that contains numbers used in calculations but not in formulas. B4 A5 D5 C10
- Circle any cell in which formulas are stored. D9 B5 A11 C4
- Write the formula stored in each cell.
 B7 _____ D11 _____
- If you change the number in cell A7 to 35, will the numbers in any other cells change?

_____ If so, which cells? _____