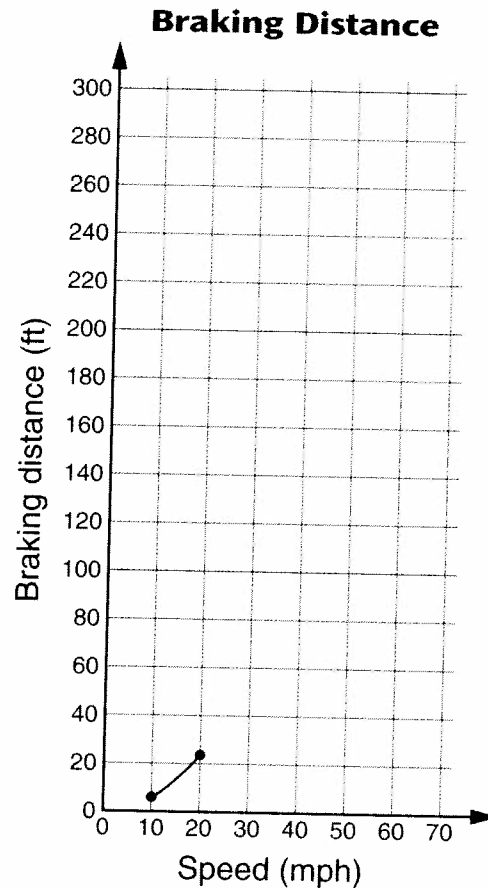
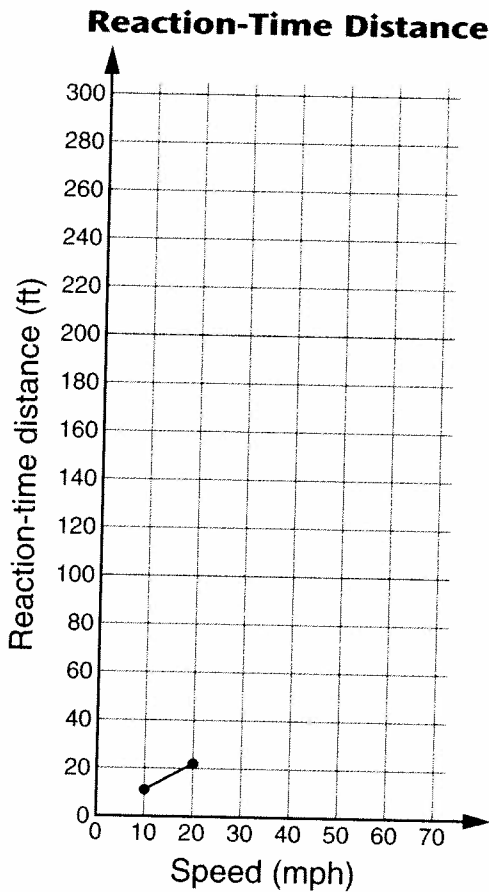


LESSON
9·7
Stopping Distance for an Automobile *cont.*


7. Use the data in the spreadsheet on *Math Masters*, page 304.
- Graph the number pairs for speed and reaction-time distance on the first grid below. Make a line graph by connecting the plotted points.
 - Graph the number pairs for speed and braking distance on the second grid below. Use a curved line to connect the plotted points.



8. How are the two graphs different?

9. Complete the statement. At speeds of 50 miles per hour or more,

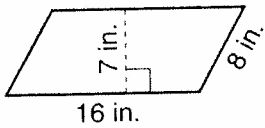
STUDY LINK
9•8

Area Problems



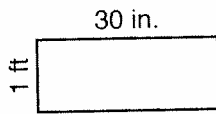
Calculate the area of each figure in Problems 1–6. Remember to include the unit in each answer.

1. parallelogram



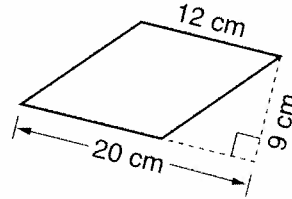
Area _____

2. rectangle



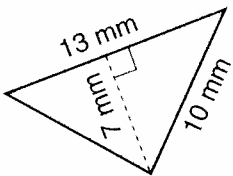
Area _____

3. parallelogram



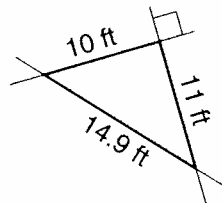
Area _____

4. triangle



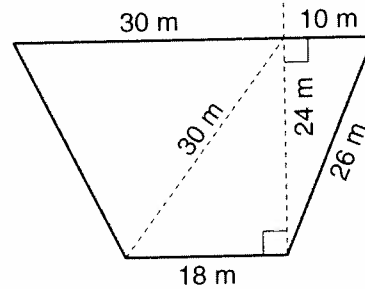
Area _____

5. triangle



Area _____

6. trapezoid

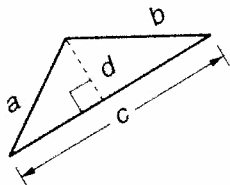


Area _____

Try This

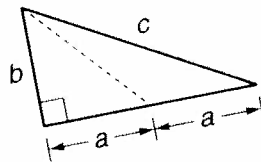
In Problems 7 and 8, all dimensions are given as variables. Write a true statement in terms of the variables to express the area of each figure.

Example:



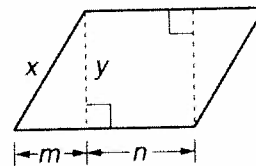
Area $\frac{1}{2} * c * d$

7.



Area _____

8.



Area _____

Practice

9. $x \div 5.3 = 12$ $x =$ _____

10. $-3.1 = -31w$ $w =$ _____

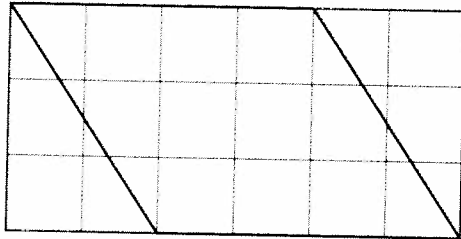
LESSON
9•8

Areas of Parallelograms



1. Do not cut out the shapes on this page. Instead cut out Parallelogram A on *Math Masters*, page 309 and follow the directions there.

Parallelogram A



Tape your rectangle in the space below.

base = _____ cm

length = _____ cm

height = _____ cm

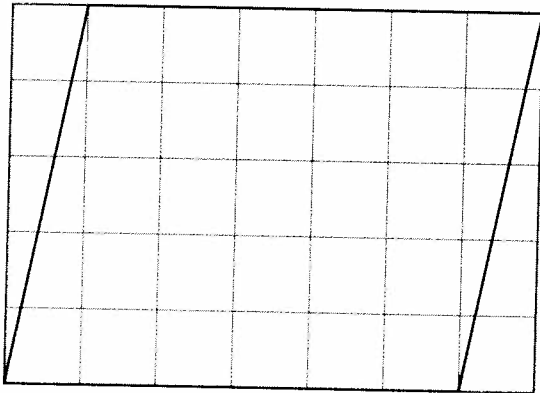
width = _____ cm

Area of parallelogram = _____ cm^2

Area of rectangle = _____ cm^2

2. Do the same with Parallelogram B on *Math Masters*, page 309.

Parallelogram B



Tape your rectangle in the space below.

base = _____ cm

length = _____ cm

height = _____ cm

width = _____ cm

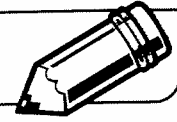
Area of parallelogram = _____ cm^2

Area of rectangle = _____ cm^2

3. Write a formula for the area of a parallelogram.

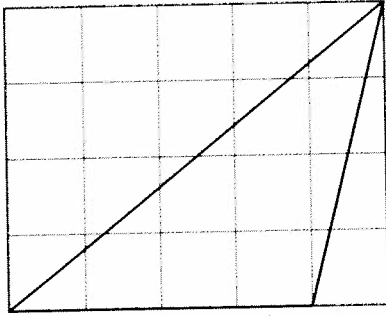
LESSON
9•8

Areas of Triangles



1. Do not cut out the triangle below. Instead cut out Triangles C and D from *Math Masters*, page 309 and follow the directions there.

Triangle C



base = _____ cm

height = _____ cm

Area of triangle = _____ cm²

Tape your parallelogram in the space below.

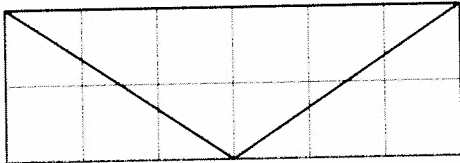
length = _____ cm

width = _____ cm

Area of parallelogram = _____ cm²

2. Do the same with Triangles E and F.

Triangle E



base = _____ cm

height = _____ cm

Area of triangle = _____ cm²

Tape your parallelogram in the space below.

length = _____ cm

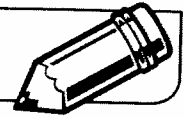
width = _____ cm

Area of parallelogram = _____ cm²

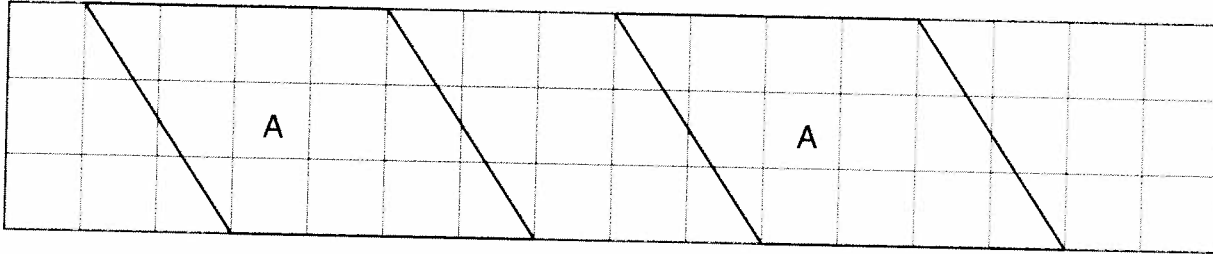
3. Write a formula for the area of a triangle.

LESSON
9•8

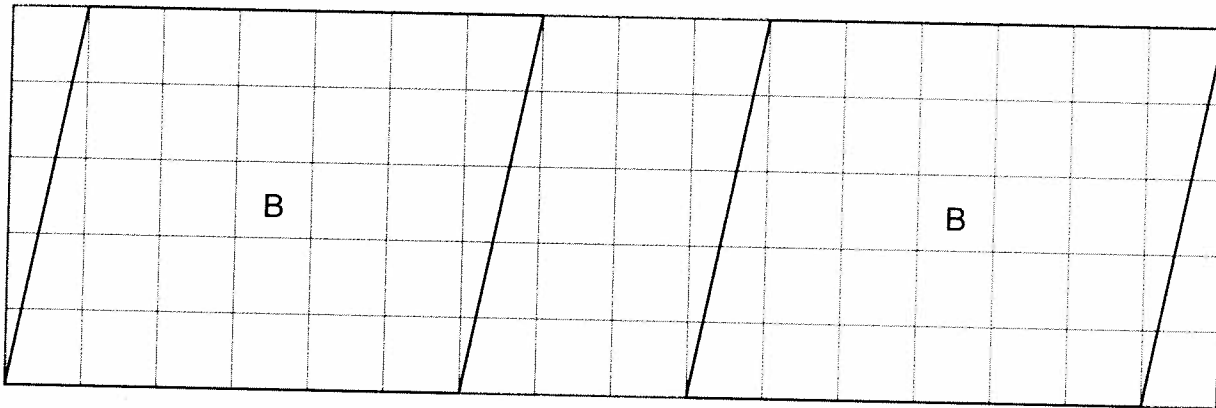
Parallelograms



Cut out Parallelogram A. (Use the second Parallelogram A if you make a mistake.) Cut it into 2 pieces and tape the pieces together to make a rectangle. Tape the rectangle onto *Math Masters*, page 307.

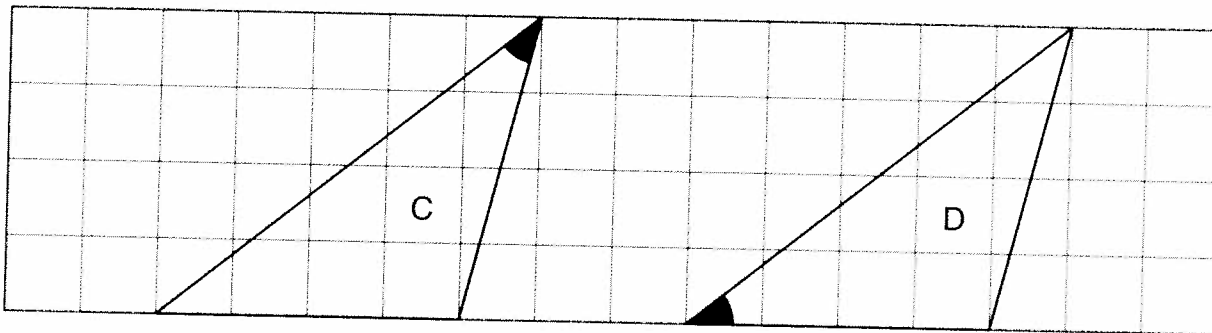


Do the same with Parallelogram B.

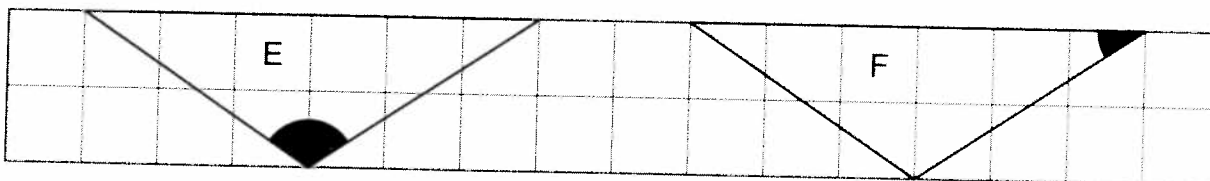


Triangles

Cut out Triangles C and D. Tape them together at the shaded corners to form a parallelogram. Tape the parallelogram onto the space next to Triangle C on *Math Masters*, page 308.

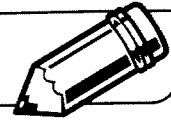


Do the same with Triangles E and F.



LESSON
9•9

Calculating the Volume of the Human Body



scale is 1 mm:1 cm

head
(sphere)

neck
(cylinder)

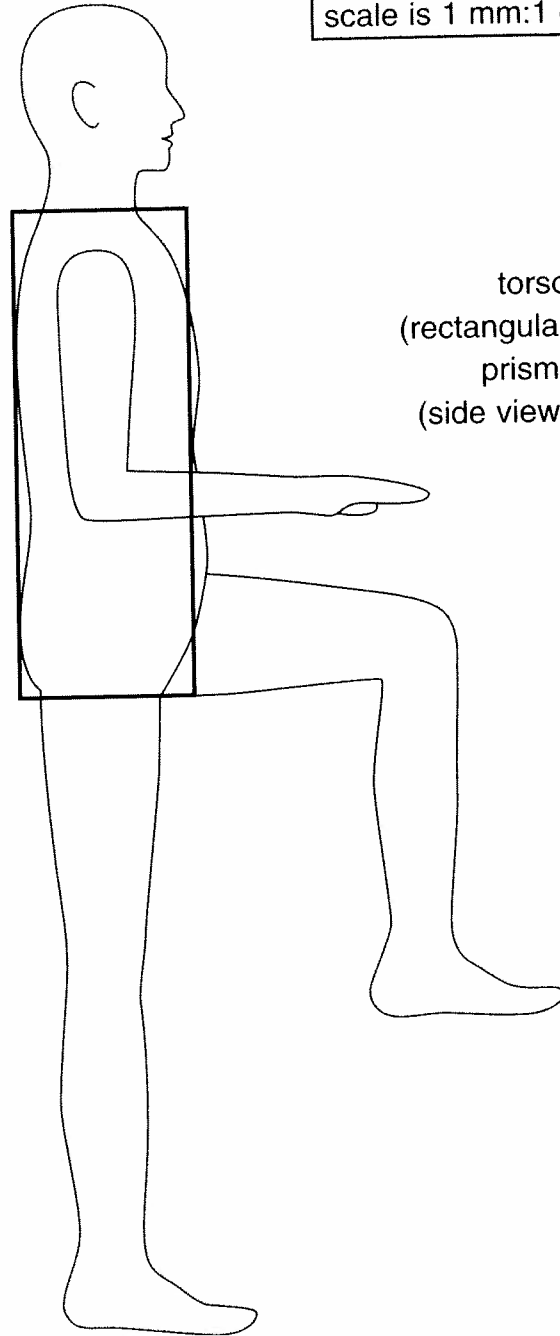
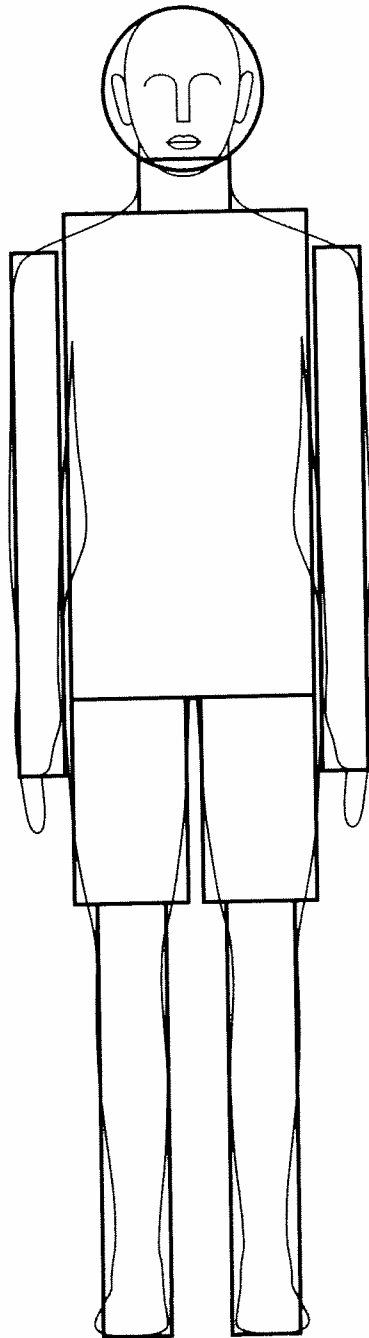
torso
(rectangular
prism)

2 arms
(cylinders)

2 upper legs
(cylinders)

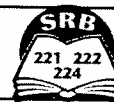
2 lower legs
(cylinders)

torso
(rectangular
prism)
(side view)



STUDY LINK
9.9

Area and Volume Problems



Area formulas

Rectangle: $A = b * h$
 Parallelogram: $A = b * h$
 Triangle: $A = \frac{1}{2} * b * h$

Volume formulas

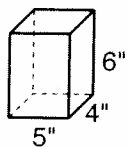
Cylinder: $V = B * h = \pi * r^2 * h$
 Rectangular prism: $V = B * h = l * w * h$
 Sphere: $V = \frac{4}{3} * \pi * r^3$

A = area
 V = volume
 B = area of base
 C = circumference
 b = length of base
 h = height
 l = length
 w = width
 r = radius

Circumference formula $C = 2\pi r$

Calculate the area or volume of each figure. Pay close attention to the units.

1.



Volume _____
 (unit)

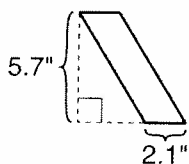
2.



diameter = 12"
 Use 3.14 for π .

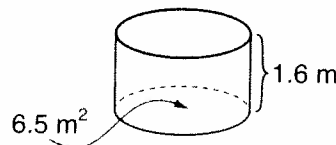
Volume _____
 (unit)

3.



Area _____
 (unit)

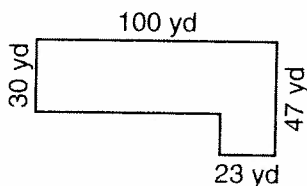
4.



Use 3.14 for π .

Volume _____
 (unit)

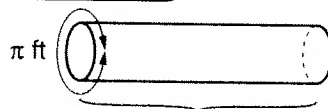
5.



Area _____
 (unit)

Try This

6.



Use 3.14 for π .

Volume _____
 (unit)

Practice

7. $0.95 \text{ m} = \text{_____ cm}$

8. $378 \text{ mm} = \text{_____ cm}$

9. $1.4 \text{ m} = \text{_____ mm}$

LESSON
9•9
Exploring Volume of Rectangular Prisms


Use what you know about area to help you find the volume of solid figures.

- Find the area of each base (B) for the partial nets on *Math Masters*, page 313. Record each base's area in the space provided on each net.
- Cut out the nets. Fold along the lines and tape the sides together to make an open box from each net.
- Fill each box by carefully layering centimeter cubes. Count the number of layers and total number of cubes needed to fill each box. Record the results in the table below.

Box	Area of base (B)	Number of layers (h)	Total number of cubes (V)
Cube	___ cm^2	___ cm	___ cm^3
Rectangular prism	___ cm^2	___ cm	___ cm^3

- Compare the area of the base (B) and the number of layers (h) with the total number of cubes (V). Explain any patterns you notice.

- Use the pattern(s) from Problem 4 to find the volume of the rectangular prisms described below.

a. $B = 40 \text{ cm}^2$; $h = 4 \text{ cm}$ $V = \text{_____} \text{ cm}^3$

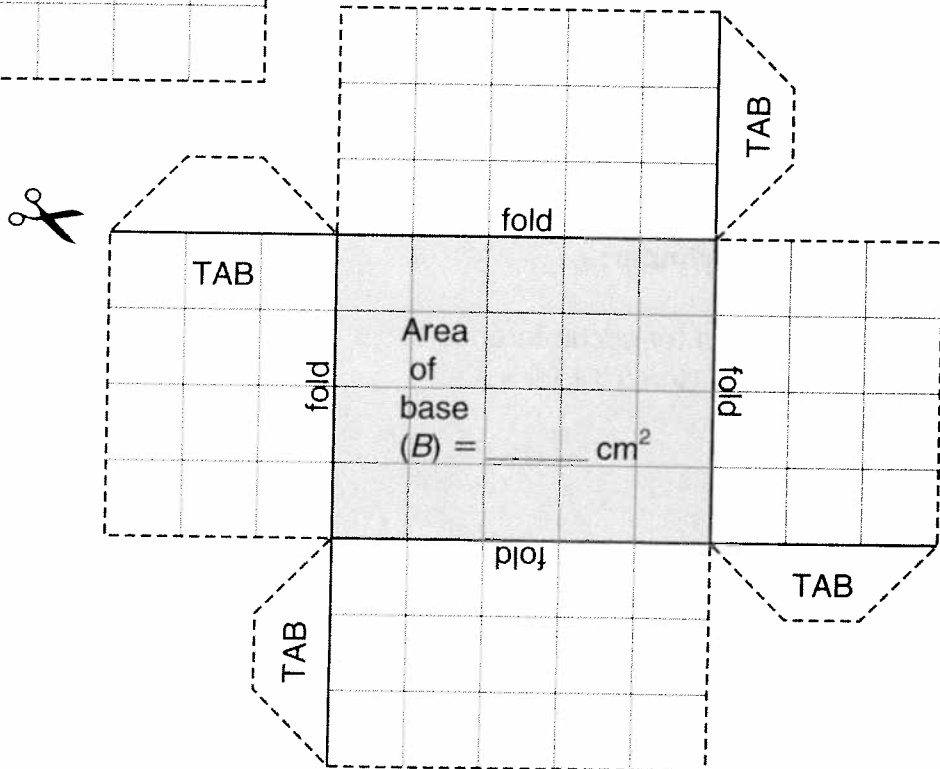
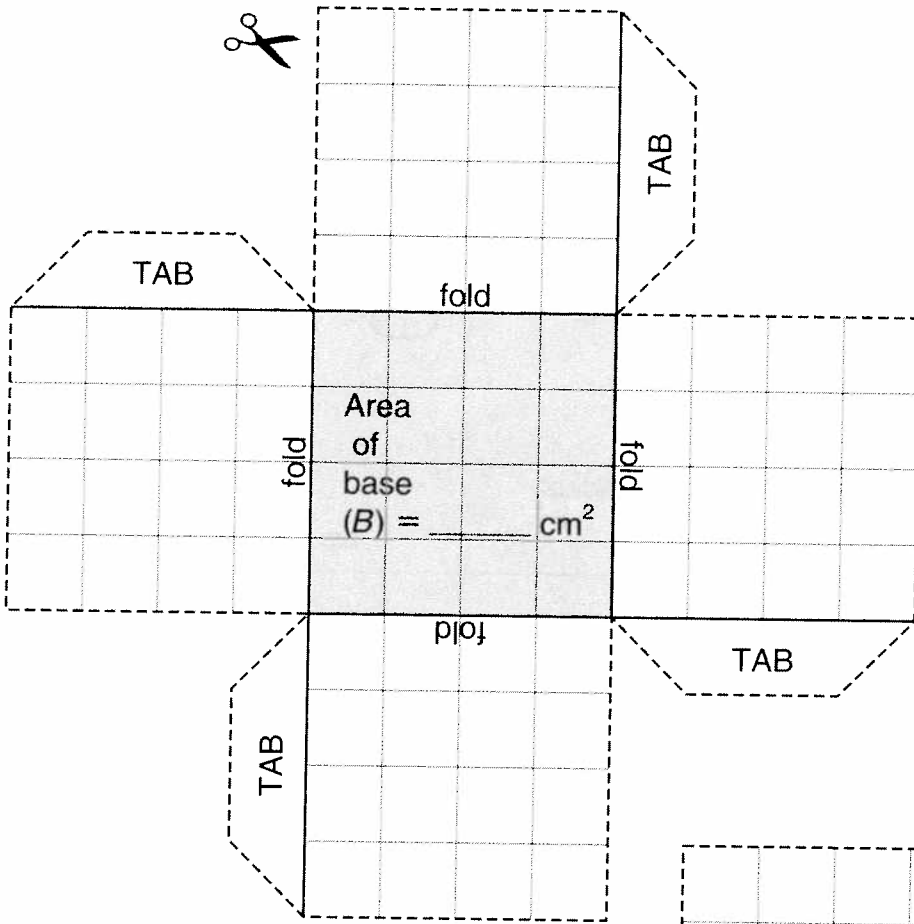
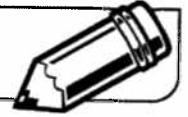
b. $B = 50 \text{ cm}^2$; $h = 8 \text{ cm}$ $V = \text{_____} \text{ cm}^3$

c. $l = 5 \text{ cm}$; $w = 9 \text{ cm}$; $h = 6 \text{ cm}$ $V = \text{_____} \text{ cm}^3$

- Suppose you know the volume of a rectangular prism is 135 cm^3 and its square base has a side measuring 3 cm . Explain how to find the height of this prism.

LESSON
9.9

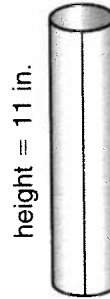
Partial Nets



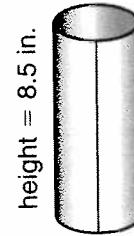
LESSON
9•9
Comparing Capacities Using Formulas


Compare the capacities of the cylinders shown at the right. The formula for the volume of a cylinder can be used to find its capacity. To find the capacity, first find the area of the circular base ($A = \pi r^2$) and multiply by the cylinder's height. Because the circumference is given, use it to find the radius; then find the area of the base. Finally, find the capacity.

circumference = 8.5 in.



circumference = 11 in.


For the taller cylinder:

1. Use the formula for circumference to find the radius.

$$C = \pi * 2r$$

 (Circumference = $\pi * 2 * \text{radius}$)

 Use 3.14 for π .

$$= \pi * 2r$$

2. Substitute the radius in the formula $A = \pi r^2$ to find the area of the base. Round this area to the nearest hundredth.

$$A = \pi * r^2$$

3. Multiply the area of the base by the cylinder's height to find the capacity.

For the shorter cylinder:

4. Use the formula for circumference to find the radius. Use 3.14 for π .

$$\pi * 2r$$

5. Substitute the radius in the formula $A = \pi r^2$ to find the area of the base. Round this area to the nearest hundredth.

$$A = \pi * r^2$$

6. Multiply the area of the base by the cylinder's height to find the capacity.

7. Which cylinder holds more?

STUDY LINK
9·10

Solving Equations by Trial and Error



Find numbers that are close to the solution of each equation.
 Use the suggested test numbers to get started.

1. Equation: $r^2 + r = 15$

r	r^2	$r^2 + r$	Compare $r^2 + r$ to 15.
3	9	12	< 15
4	16	20	> 15
3.5	12.25	15.75	> 15

My closest solution _____

2. Equation: $x^2 - 2x = 23$

x	x^2	$2x$	$x^2 - 2x$	Compare $x^2 - 2x$ to 23.
6	36	12	24	> 23
5	25	10	15	< 23
5.5	30.25	11	19.25	< 23

My closest solution _____

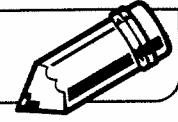
Practice

3. $56 - 42.52 =$ _____

4. $23.5 - 5.88 =$ _____

LESSON
9•10

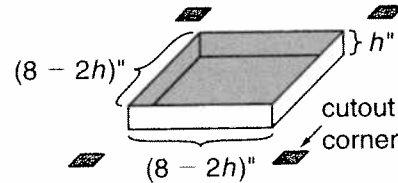
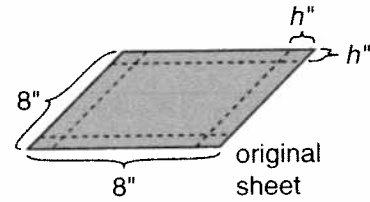
A Box Problem



Suppose you have a square piece of cardboard that measures 8 inches along each side. To construct an open box out of the cardboard, you can cut same-size squares from the 4 corners of the cardboard and then turn up and tape the sides.

1. José cut out small square corners to make his box.
Amy cut out large square corners to make her box.

- a. Whose box was taller? _____
b. Whose box had a greater area of the base? _____



Box made by cutting out square corners and folding up sides

The volume of the box depends on the size of the squares cut from the corners.

2. Find the dimensions of the box with the greatest possible volume. Use trial and error to solve the problem. Keep a record of your results in the spreadsheet below.
- a. Three test values for h (the height of the box) are listed in Column A. Complete rows 4, 5, and 6.

Boxes				
	A	B	C	D
1	Problem: Find the length that maximizes the box volume.			
2	box height (in.)	box length, width (in.)	base area of box (in. ²)	volume of box (in. ³)
3	h	$8 - 2h$	$(8 - 2h)^2$	$(8 - 2h)^2 * h$
4	1	6		36
5	2			
6	3			
7				
8				
9				

- b. Use the spreadsheet results to select new test values for h that are likely to give a box of greater volume. Complete the statement below.

The box that I found with the greatest volume has a height of _____ inches and a volume of _____ cubic inches.

STUDY LINK
9-11

Using Formulas



Each problem below states a formula and gives the values of all but one of the variables in the formula. Substitute the known values for the variables in the formula and then solve the equation.

1. The formula $C = \frac{5}{9} * (F - 32)$ may be used to convert between Fahrenheit and Celsius temperatures.

- a. Convert 77°F to degrees C.

Equation _____

Solve.

$$77^{\circ}\text{F} = \text{_____}^{\circ}\text{C}$$

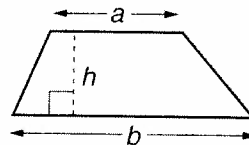
- b. Convert 50°C to degrees F.

Equation _____

Solve.

$$50^{\circ}\text{C} = \text{_____}^{\circ}\text{F}$$

2. The formula for the area of a trapezoid is $A = \frac{1}{2} * (a + b) * h$.



- a. Find the area (A) of a trapezoid if $a = 7$ cm, $b = 10$ cm, and $h = 5$ cm.

Equation _____

Solve.

Area _____
(unit)

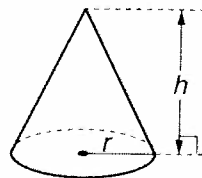
- b. Find the height (h) of a trapezoid if $a = 6.5$ inches, $b = 5.5$ inches, and $A = 90$ inches².

Equation _____

Solve.

Height _____
(unit)

3. The formula for the volume of a cone is $V = \frac{1}{3} * \pi * r^2 * h$.
Use 3.14 for π .



- a. Find the volume (V) of a cone if $r = 2$ inches and $h = 9$ inches.

Equation _____

Solve.

Volume _____
(unit)

- b. Find the height (h) of a cone if $r = 3$ cm and $V = 94.2$ cm³.

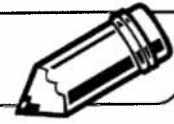
Equation _____

Solve.

Height _____
(unit)

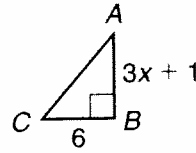
LESSON
9•11

Perimeter and Area Problems



Study the example. Then solve the problems.

Example: The area of triangle CBA is 21 square inches.
What is the length of side BA ?



- Solution**
- Write the formula for the area of a triangle.
 - Substitute the dimensions in the formula.
 - Solve the equation.

$$A = \frac{1}{2} * b * h$$

$$21 = \frac{1}{2} * 6 * (3x + 1)$$

$$21 = 3 * (3x + 1)$$

$$21 = 9x + 3$$

$$18 = 9x, \text{ so } 2 = x.$$

- Answer the question. $3x + 1 = (3 * 2) + 1$, or 7
Side AB is 7 inches long.
- Check the answer: Area = $\frac{1}{2} * 6 * 7 = \frac{1}{2} * 42 = 21 \text{ in.}^2$

- The area of rectangle $RPQT$ is 14 ft^2 . Find the length of side RP .

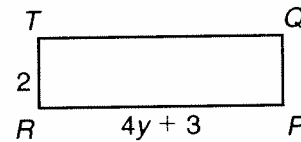
Formula: Area = _____

Substitute _____ = _____

Solve _____

Length of \overline{RP} _____

Check _____



- The area of parallelogram $FLOW$ is 15 in.^2 . Find the length of side FL .

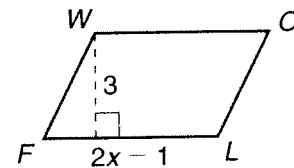
Formula: Area = _____

Substitute _____ = _____

Solve _____

Length of \overline{FL} _____

Check _____



- The perimeter of triangle MON is 29 cm. Find the length of each side.

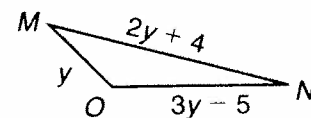
Formula: Perimeter = _____

Substitute _____ = _____

Solve _____

Length of \overline{NM} _____ \overline{ON} _____ \overline{MO} _____

Check _____



STUDY LINK
9·12

Pythagorean Theorem



Mentally find the positive square root of each number.

1. $\sqrt{144} = \underline{\hspace{2cm}}$

2. $\sqrt{200^2} = \underline{\hspace{2cm}}$

3. $\sqrt{900} = \underline{\hspace{2cm}}$

4. $\sqrt{0.16} = \underline{\hspace{2cm}}$

5. $\sqrt{\frac{25}{121}} = \underline{\hspace{2cm}}$

6. $\sqrt{10,000} = \underline{\hspace{2cm}}$

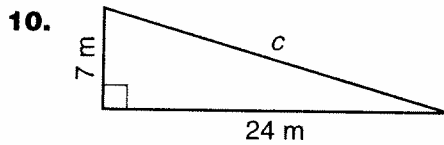
Use a calculator to find each square root. Round to the nearest hundredth.

7. $\sqrt{12} = \underline{\hspace{2cm}}$

8. $\sqrt{51} = \underline{\hspace{2cm}}$

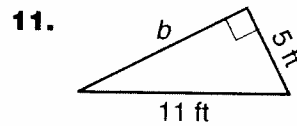
9. $\sqrt{63} = \underline{\hspace{2cm}}$

Use the Pythagorean theorem to find each missing length. Round your answer to the nearest tenth.



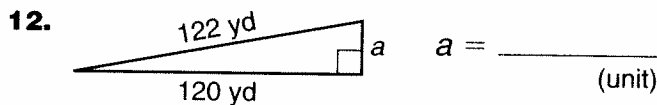
$$c = \underline{\hspace{2cm}}$$

(unit)



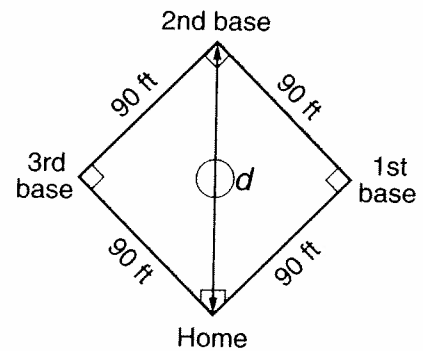
$$b = \underline{\hspace{2cm}}$$

(unit)



13. Find the distance (d) from home plate to second base.

$$d = \underline{\hspace{2cm}}$$
 ft



Practice

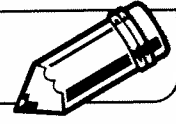
Simplify.

14. $2[9(6 - 5)] = \underline{\hspace{2cm}}$

15. $5 + 3 * 4 - 8 + 2 * 7 = \underline{\hspace{2cm}}$

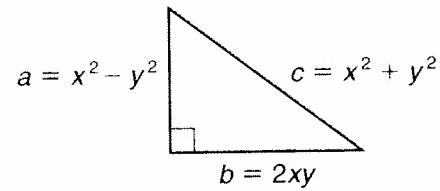
LESSON
9•12

Pythagorean Triples



Sets of positive integers that are solutions of the equation $a^2 + b^2 = c^2$ are Pythagorean triples. The smallest Pythagorean triple is 3, 4, 5.

You can find Pythagorean triples by choosing any 2 positive integers, x and y , where $x > y$, and using the formulas $a = x^2 - y^2$, $b = 2xy$, and $c = x^2 + y^2$.



- Study the example in the table below. Then use the formulas to complete the table. Use the Pythagorean theorem to make sure each triple works.

Formulas		Leg a	Leg b	Leg c	Pythagorean triple
x	y	$x^2 - y^2$	$2xy$	$x^2 + y^2$	
2	1	$2^2 - 1^2 = 3$	$2(2)(1) = 4$	$2^2 + 1^2 = 5$	3, 4, 5
3	1				
3	2				
4	1				
4	3				
5	4				
6	5				

- Use any patterns you notice in the table above and a trial-and-error strategy to help you find the values of x and y that generate each triple.

a. 21, 20, 29 $x =$ _____ $y =$ _____

b. 27, 36, 45 $x =$ _____ $y =$ _____



1. Simplify the following expressions by combining like terms.

a. $4x + 3x =$ _____

b. $3x + 7 + x =$ _____

c. $4 * (x + 2) + 2x - 6 =$ _____

d. $(x + 3) * 2 - 2x =$ _____

2. Liani simplified the expression $8(x + 10)$ as $(8 * x) + 10$. What did she do wrong? Explain her mistake and show the correct way to solve the problem.

3. Solve each equation. Show your work on the back of this sheet.

a. $3x - 4 = 4x + 6$ _____

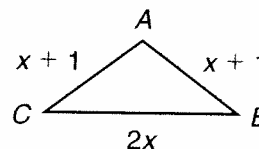
b. $5 * (2 - 6) = 4g$ _____

c. $3(2y - 3) = 15$ _____

d. $\frac{(2x - 1)}{3} = 9$ _____

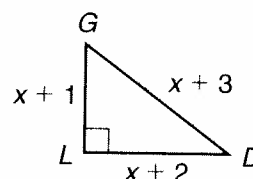
4. The perimeter of triangle ABC is 18 inches. What is the length of each side?

AB _____ BC _____ AC _____

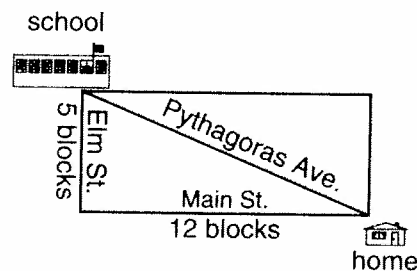


5. The perimeter of right triangle GLD is 12 centimeters.

What is the area of the triangle? _____



6. Toshi often walks to school along Main Street and Elm Street. If he were to take Pythagoras Avenue instead, how many fewer blocks would he walk? _____

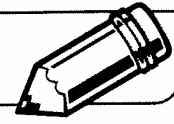


Practice

7. $28 \overline{)42} =$ _____

8. $161 \div 92 =$ _____

9. $200 \overline{)120} =$ _____

LESSON
9•13**An Indirect Measurement Problem**

Work with 3 other students. Your teacher has taped a target on the wall. You will use an indirect method to determine the height of the target above the floor.

Study the diagram shown below. Each student has a special job.

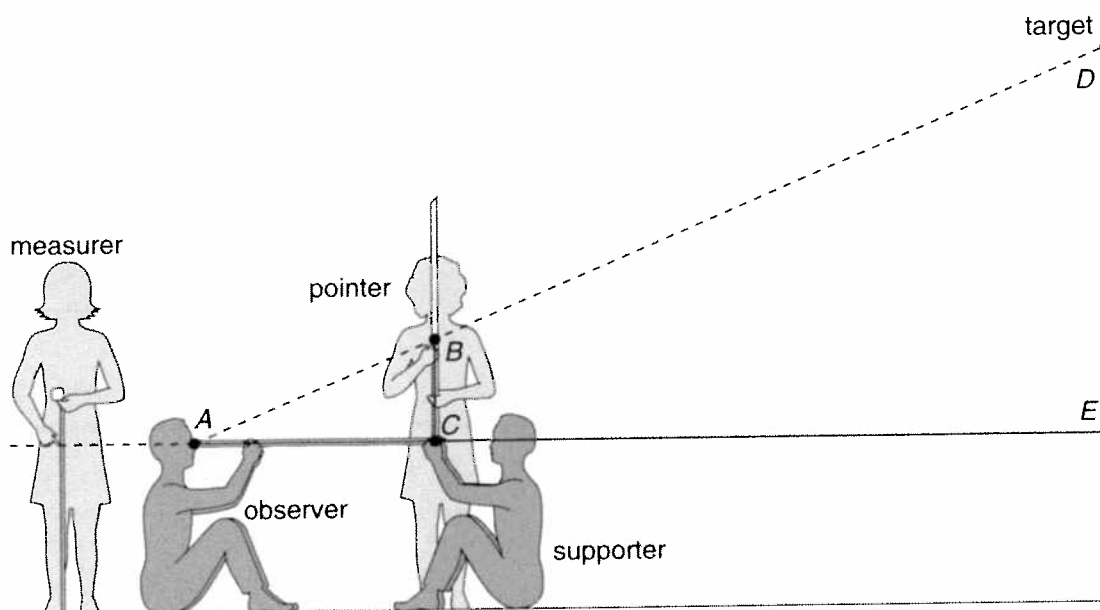
Observer: Sit on the floor and face the target. Sit about 4.5 to 6 meters from the target.

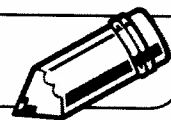
Supporter: You and the observer hold a meterstick so it is at the observer's eye level. Make sure the meterstick is parallel to the floor.

Pointer: Take a second meterstick and place the 0 end on top of the end of the meterstick that the supporter is holding. The supporter holds the ends of the sticks together. Make sure to hold the meterstick vertically so angle ACB is approximately a right angle (90°).

Observer: Hold the end of the meterstick (point A) near your eye and look at the target (point D). Instruct the pointer to slide a finger up or down the vertical meterstick until the finger appears to point to the target (point D). Record the length of \overline{BC} .

Measurer: Measure the height above the floor of the observer's meterstick (the height of \overline{AC} above the floor). Also measure the distance from the observer's eye to the wall (the length of \overline{AE}).



LESSON
9·13**An Indirect Measurement Problem *cont.***

1. Record your measurements.

$$AC = 100 \text{ cm}$$

$$AE = \text{_____ cm (distance from observer's eye to wall)}$$

$$BC = \text{_____ cm}$$

$$\text{Distance from observer's eye to floor} = \text{_____ cm}$$

2. Draw sketches of triangles ACB and AED that include your measurement information.

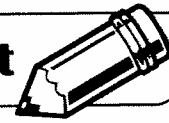
3. Triangles ACB and AED are similar figures.

What is the size-change factor for these figures? _____

Use the size-change factor to calculate the length of \overline{DE} .

$$DE = \text{_____ cm}$$

4. What is the height of the target above the floor? _____

LESSON
9•13
Proportions and Indirect Measurement


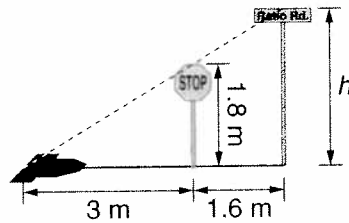
One way to solve an indirect measurement problem is to write a proportion. Study the example below.

Example:

A road sign casts a shadow that is 4.6 meters long. A stop sign near the road sign casts a shadow that is 3 meters long. The road sign and its shadow form 2 legs of a right triangle that are similar to the 2 legs of a right triangle formed by the stop sign and its shadow.

Find the height of the road sign.

Use the diagram to write a proportion involving the corresponding sides of the triangles.



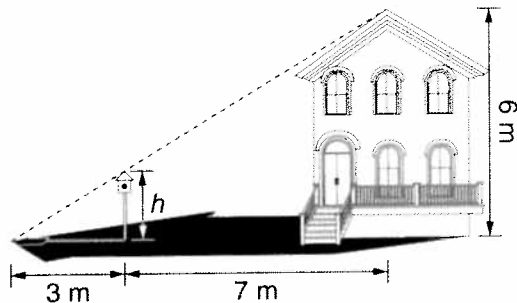
$$\begin{array}{l} \frac{\text{stop sign's height}}{\text{road sign's height}} \rightarrow \frac{1.8}{h} = \frac{3}{4.6} \leftarrow \frac{\text{length of stop sign's shadow}}{\text{length of road sign's shadow}} \end{array}$$

Use cross products to write an equation. $3h = (1.8)(4.6)$

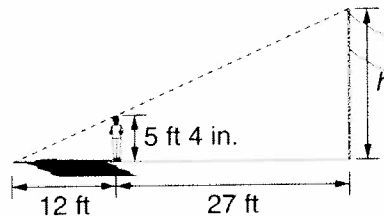
$$\begin{array}{l} \text{Solve.} \quad \frac{3h}{3} = \frac{8.28}{3} \\ h = 2.76 \end{array}$$

The height of the road sign is 2.76 meters.

The triangles in Problems 1 and 2 are similar. Solve each problem by writing and solving a proportion.

1.


Height of birdhouse = _____ (unit)

2.


Height of flagpole = _____ (unit)